Course 16
Geometric Data Structures for Computer Graphics

Quadtrees

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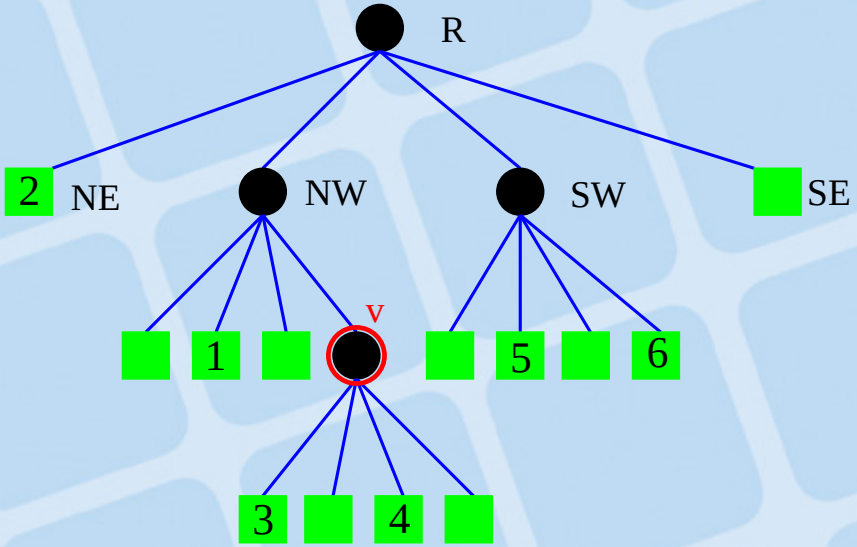
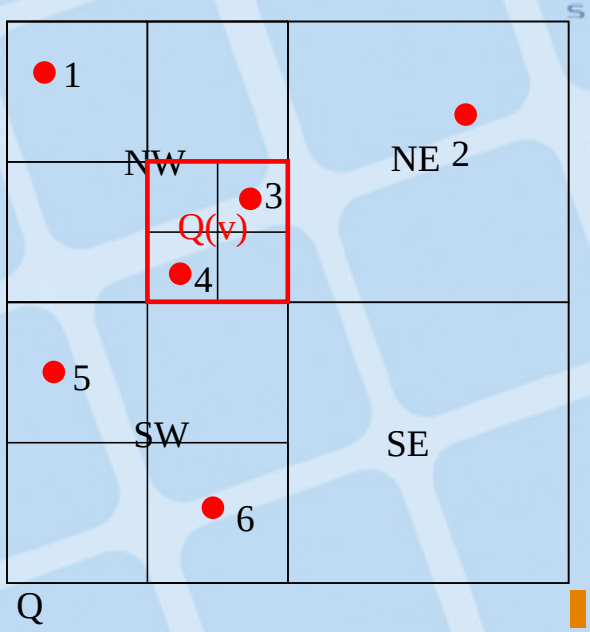
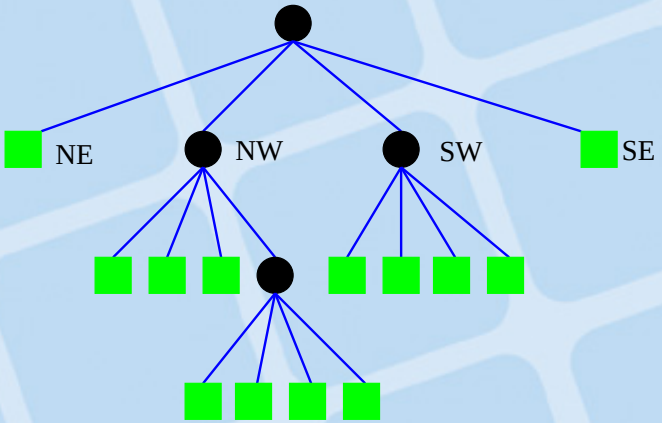
Quadtree of a point set

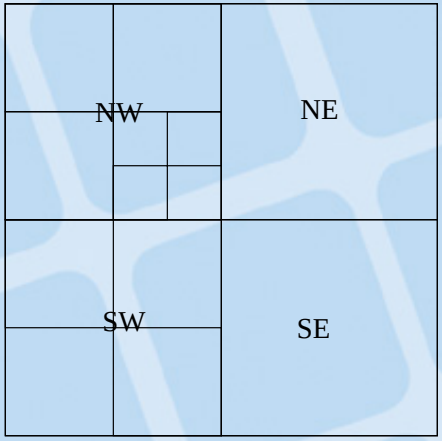




- Set of points, initial sqaure Q and root R of the tree
- Subdivision into quadrants in counterclockwise order
- Recursively, until sqaure has ≤ 1 objects
- Node v represents square $Q(v)$
- Recursive construction of the tree: given points/initial square

Definition Quadtree





- Rooted tree
- Internal nodes have 4 children
- Every node represents a square
- Children represent subsquares of the square
- Geometric data of squares (leaves): Points, Lines, Rectangles, Ellipses
- *Octree* $\Rightarrow$ 8 children, Boxes, higher Dimensions

Properties

- Quadtree of depth d with n points
- Number of nodes: $O(d\,n)$
  - Number of leaves: $3 \times$ # *Internal nodes* $+ 1$
  - At every depth only n internal nodes
- Construction: $O(d\,n)$ time
  - Every depth in the recursive construction
  - Distribution of points: Linear in the number of points
- Depth of the quadtree depends on distances of objects:
  - Let c be the distance of the closest pair
  - Let s be the side length of the initial Q
  - Depth $d \leq \log(s/c) + \frac{3}{2}$
- Balancing depend on objects

Application Nearest Neighbors

- Compute List of Nearest Neighbors of a query point q
- Idea: Observe Neighboring quadrants recursively
  - Find quadrant of query point q in $O(d)$ time
  - Build Priority-Queue P: Visited squares sorted by distance in $O(d \log d)$ time
  - Iteration
    - Take first (closest) square/object O of P
    - Object: $\Rightarrow$ report
    - Square: $\Rightarrow$ insert subsquares or single object into P
    - Delete O out of P
    - Repeat until P is empty
- Time: $O(n \log n)$

Nearest Neighbor Applet by F. Brabec and H. Samet