The Expected Running Time of Hierarchical Collision Detection

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Abstract

We propose a model to analyze the expected running time of hierarchical collision detection that utilizes bounding volume hierarchies (BVHs). It depends on two characteristic parameters: the overlap of the root BVs and the *BV diminishing factor* within the hierarchies. We show that the average running time is in $O(n)$ or even in $O(\lg n)$ for realistic cases, where n is the number of leaves. Our theoretical analysis is verified by practical experiments.

1 Hierarchical Collision Detection

In practice, bounding volume hierarchies (BVHs) have proven to be a very efficient data structure in the area of collision detection, even for (reduced) deformable models [James and Pai 2004].

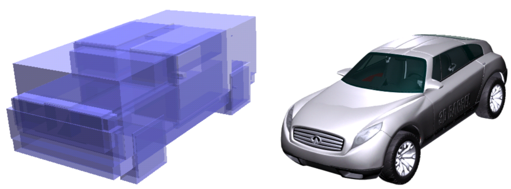

Figure 1: BVH of a model (courtesy www.3dbarrel.com).

Most collision detection approaches (conceptually) traverse a *bounding volume test tree* (see Figure 2). To estimate the time required for a collision query, the well-known cost function

$$T = N_v C_v + N_p C_p + N_u C_u + C_i$$

has been proposed [Weghorst et al. 1984; Gottschalk et al. 1996]. In an asymptotic analysis, N_v, the number of overlap tests, defines the running time. While it is obvious that $N_v = n^2$ in the worst case, it has long been noticed by many researchers that this number seems to be linear or even logarithmic for most practical cases. However, until now there is no rigorous analysis for the expected running time of a BVH traversal.

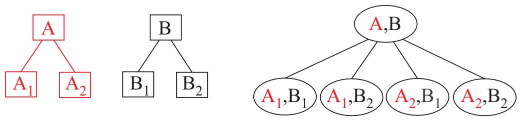

Figure 2: The conceptual BV test tree (BVTT).

2 Complexity Analysis

The general approach of our analysis is as follows. First, for each level l of the BVTT, we estimate the number $\tilde{N}_v^{(l)}$ of nodes whose x-overlap is > 0. That means, given a node and its corresponding BVs $A_i^{(l)}, B_j^{(l)}$, we test whether the overlap $o_{ij}^{(l)}$ of the BVs $A_i^{(l)}$ and $B_j^{(l)}$, when projected onto the x axis, is larger than zero. Second, we determine the conditional probability that the BV pair $(A_i^{(l)}, B_j^{(l)})$ on level l overlaps

$$p^{(l)} := Pr[A_i^{(l)} \cap B_j^{(l)} \neq \varnothing \mid A \cap B \neq \varnothing \wedge o_{ij}^{(l)} > 0]$$

where (A,B) is the parent node of $(A_i^{(l)}, B_j^{(l)})$ in the BVTT. In contrast to what one might think, it turns out that we do not need individual probabilities per BV pair.

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probability	$\tilde{N}_v$
$p^{(l)} < 1/4$	$O(1)$
$p^{(l)} = 1/4$	$O(\lg n)$
$p^{(l)} = 1/2$	$O(n)$
$p^{(l)} = 3/4$	$O(n^{1.58})$
$p^{(l)} = 1$	$O(n^2)$

Table 1: Effect of the probability $p^{(l)}$ on the expected running time.

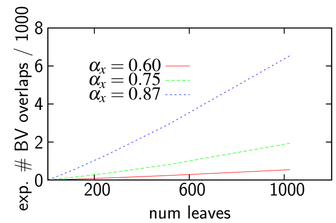

Figure 3: Overall expected number of overlapping BV pairs $\tilde{N}_v(n)$, with root $o_{ij}^{(0)} = 0.4$, $\alpha_x = \alpha_y = \alpha_z$.

Let us assume a constant *BV diminishing factor* throughout the hierarchy, i.e.,

$$a'_x = \alpha_x a_x, \qquad a'_y = \alpha_y a_y, \qquad a'_z = \alpha_z a_z,$$

where a and a' are the side lengths of father and child BV, resp. Then, by way of 2-dimensional Minkowski sums, it is easy to see that $p^{(l)} = \alpha_y \alpha_z$. Let X_l denote the number of visited nodes on level l in the BVTT. Then, its expected number is

$$E(X_l) = \tilde{N}_v^{(l)} \cdot \prod_{i=1}^{l} p^{(l)} = \tilde{N}_v^{(l)} \cdot \alpha_y^l \alpha_z^l.$$

where we only assume that the two root BVs overlap (i.e., $p^{(0)} = 1$). Overall, the expected total number of visited nodes is

$$\tilde{N}_v = \sum_{l=1}^{d} E(X_l) \leq \sum_{l=1}^{d} 4^l \cdot \alpha_y^l \alpha_z^l \in O(n^{\lg 4\alpha_y\alpha_z}),$$

where $d = \log_4(n^2) = \lg(n)$ is the depth of the BVTT (equaling the depth of the BVHs). Table 1 shows the running time for various values of $p^{(l)}$. Clearly, $\tilde{N}_v^{(l)} \leq 4^l$ is not a tight bound. Therefore, exploiting some geometric arguments, we can determine a probability distribution for the expected value of $o_{ij}^{(l)}$, depending on the root overlap $o_{ij}^{(0)}$ as well as on which child we go into by traversing the BVTT (details are given in our suppl. material). Fig. 3 shows $\tilde{N}_v(n)$ for different parameters.

3 Results and Conclusion

We have presented an approach to analyze simultaneous BV hierarchy traversals that provides a better understanding of the observed performance of hierarchical collision detection than in the past. Further, we have determined the average number of BV pairs visited for synthetic and real-world BVHs in a series of experiments. For all our models, the running time behaved always like $O(\lg n)$.

References

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