

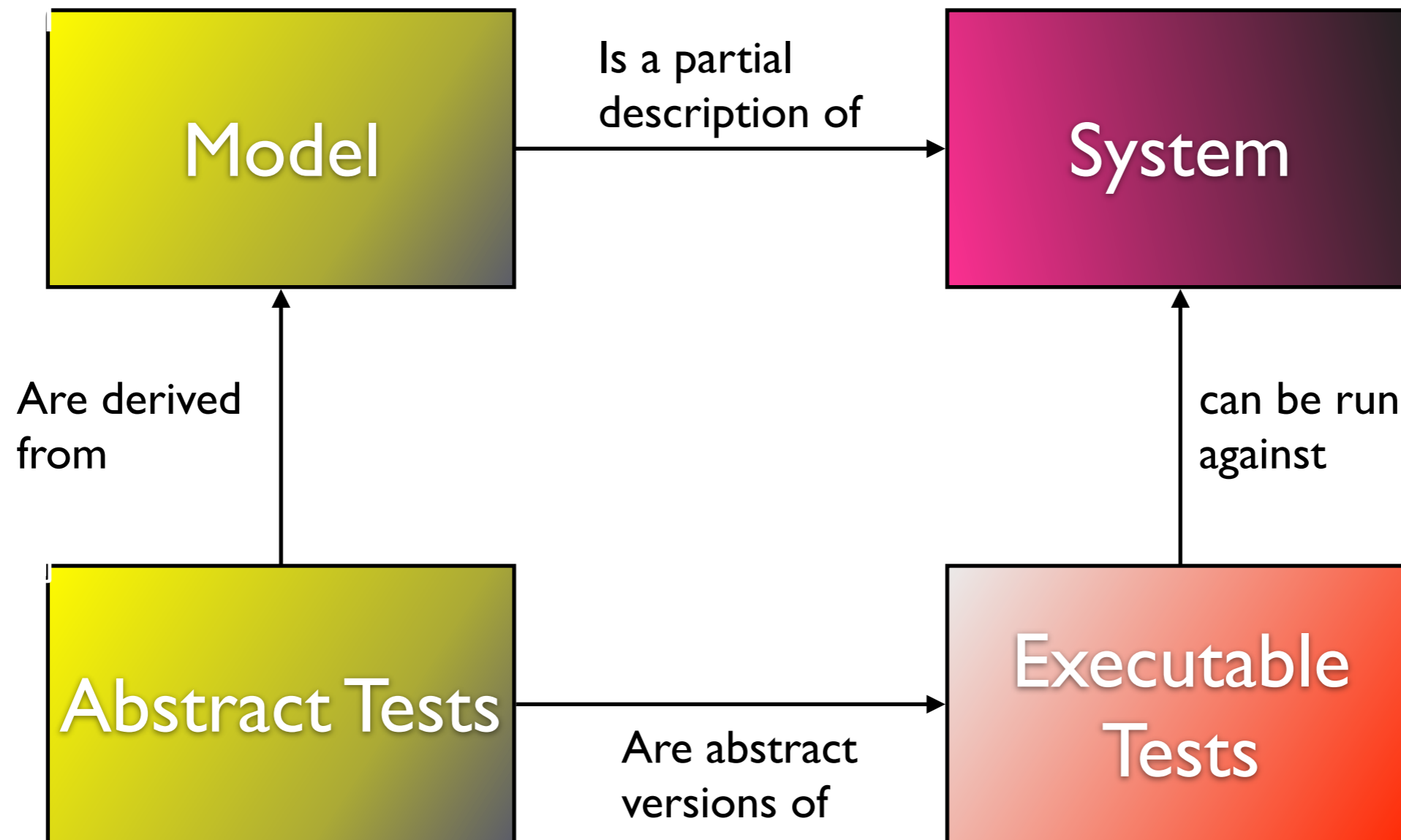
# Testing Infinite State Systems – Mathematical Foundations and Concrete Algorithms

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# Model-Based Testing

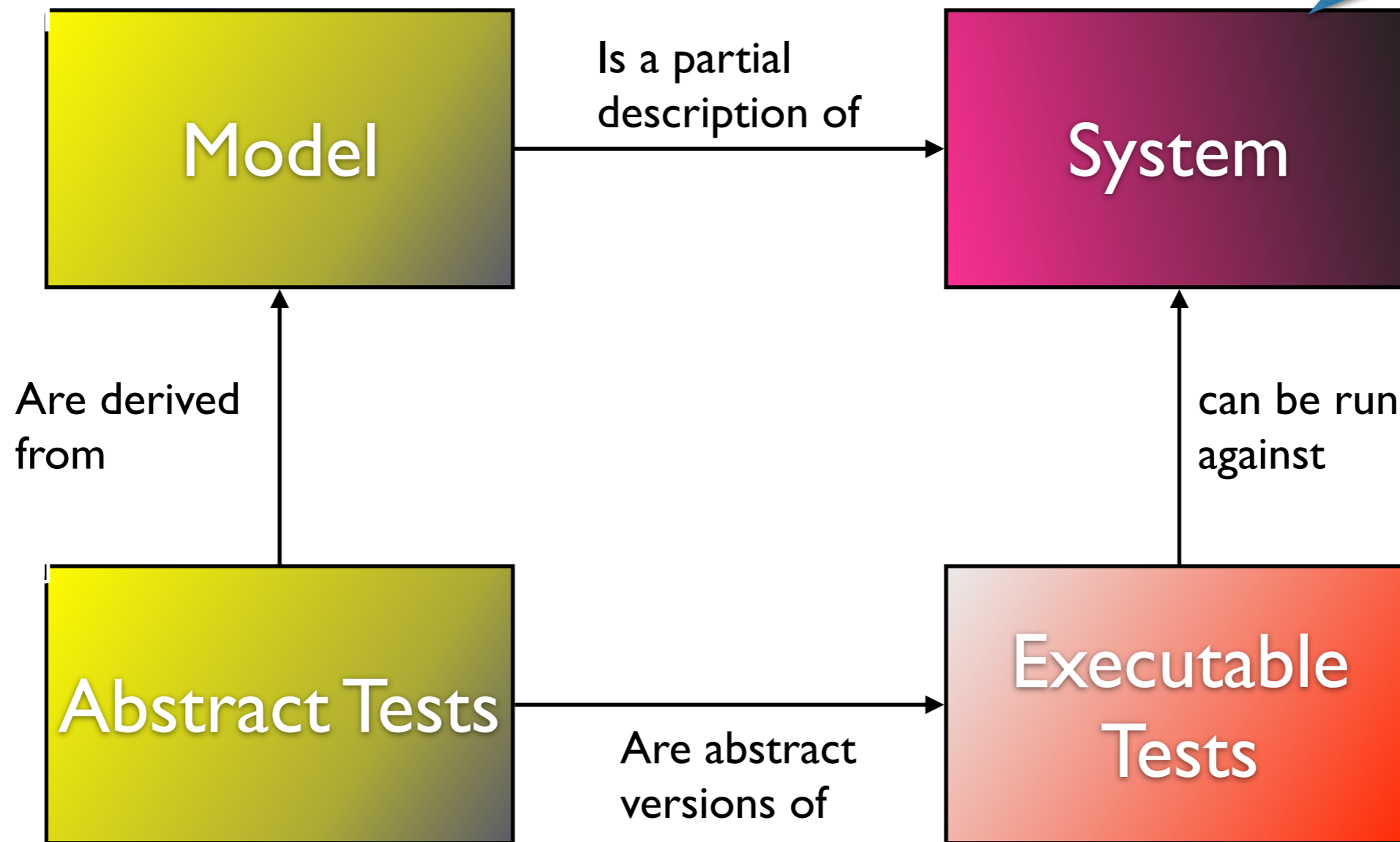
- Model-based testing (MBT) as defined in Wikipedia
  - “**Model-based testing** is an application of **model-based** design for designing and optionally also executing artifacts to perform software **testing** or system **testing**. **Models** can be used to represent the desired behavior of a System Under Test (SUT), or to represent **testing** strategies and a test environment.”

# MBT-Paradigm



# MBT-Paradigm

被測試的系統





# MBT-Paradigm

被測試的系統

FSM

Model

Is a partial  
description of

System

Are derived  
from

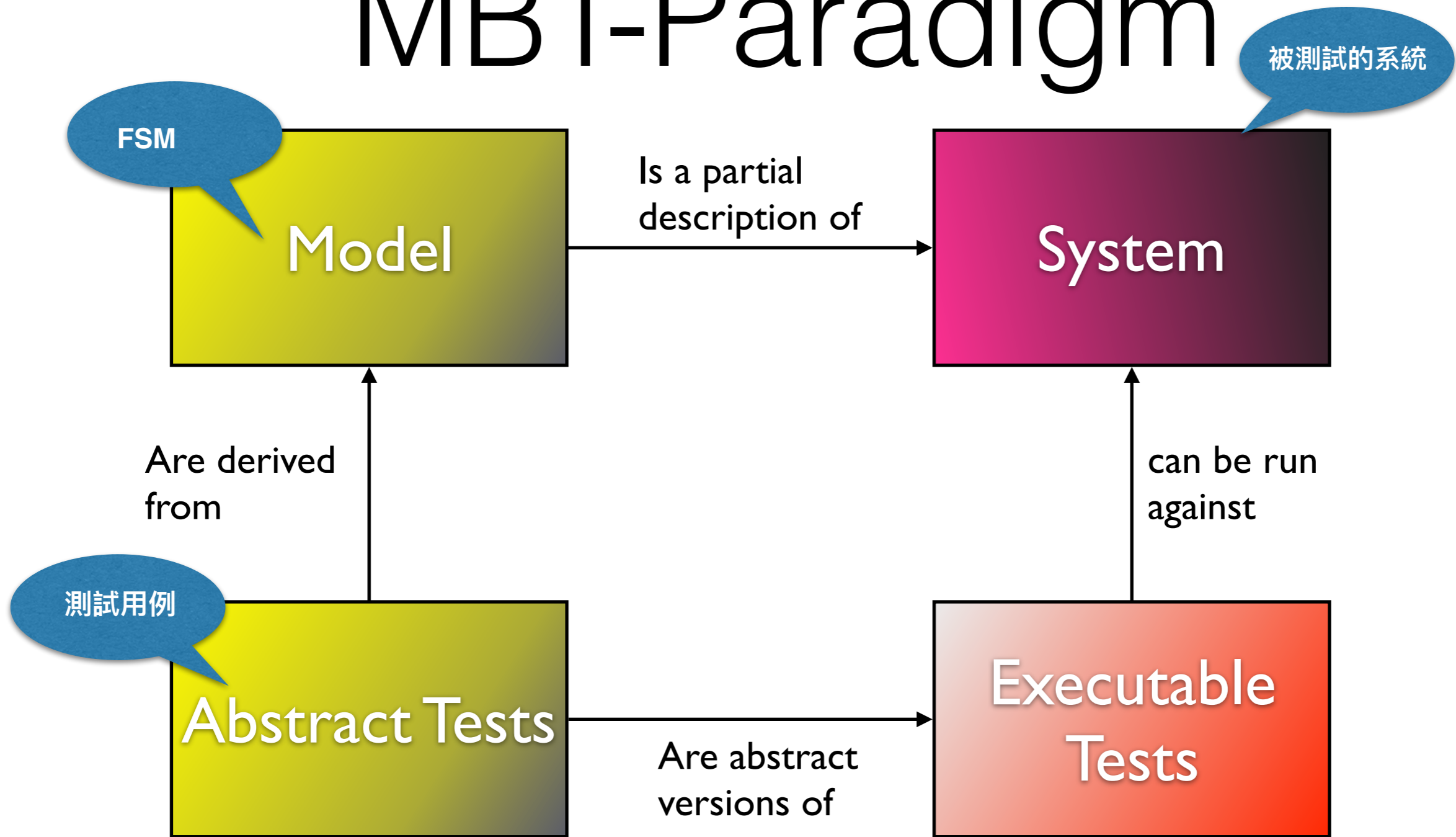
Abstract Tests

Are abstract  
versions of

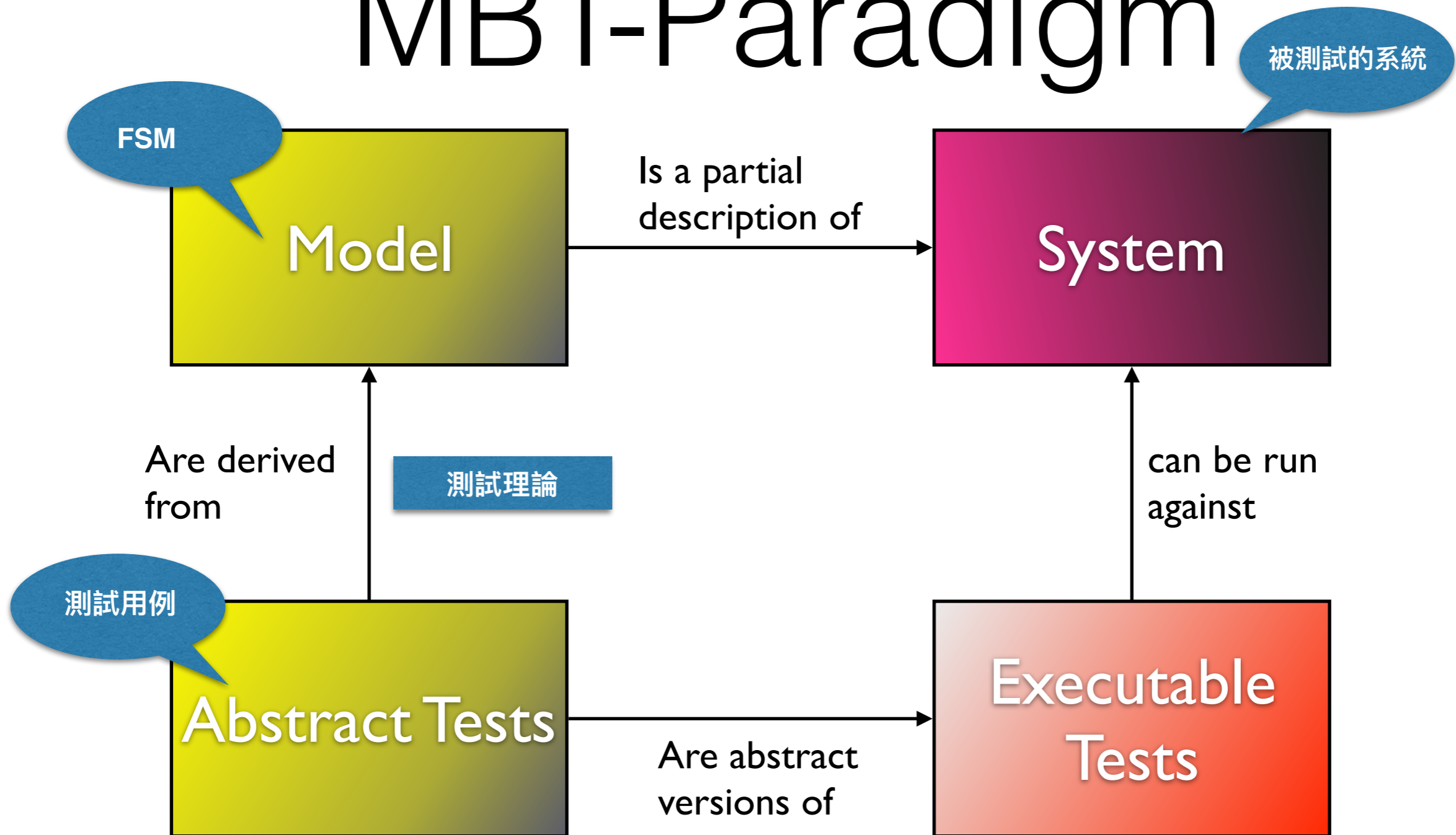
Executable  
Tests

can be run  
against

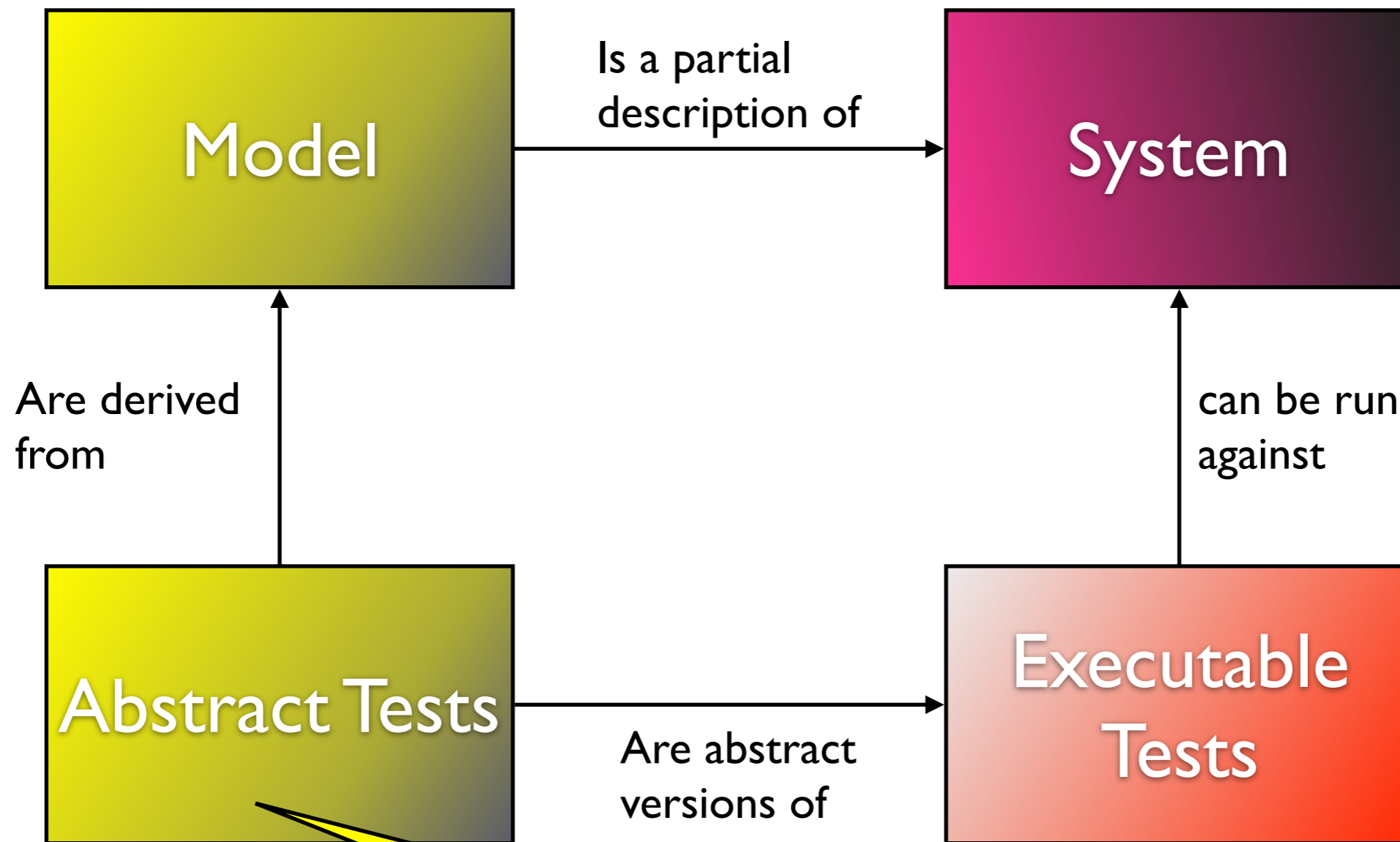
# MBT-Paradigm



# MBT-Paradigm

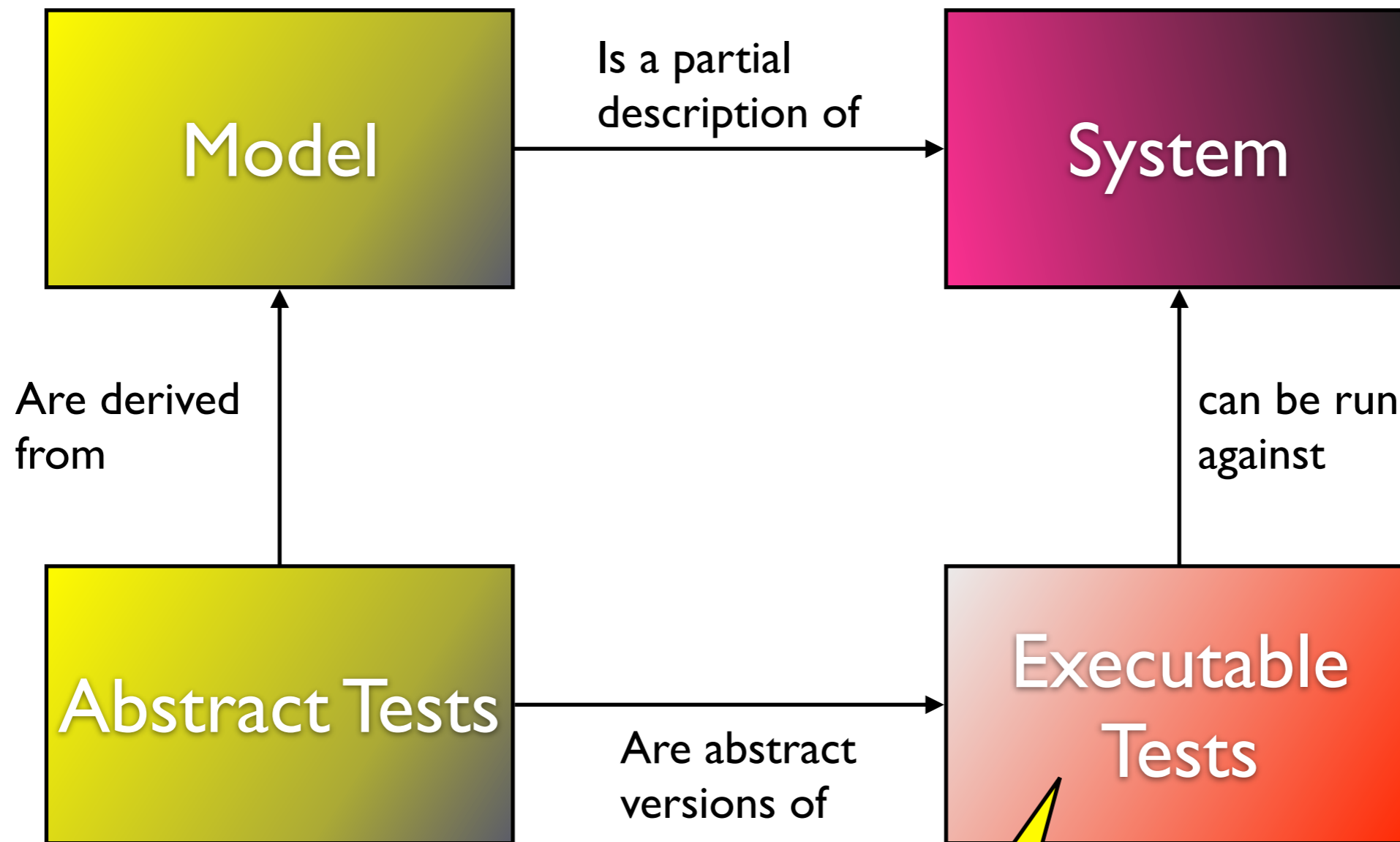


# MBT-Paradigm



We also call these **symbolic tests**, since they can be represented by logical formulas

# MBT-Paradigm



We also call these **test procedures**, as is suggested by several standards, such as RTCA DO-178B

# Testing Theories

$\mathcal{F}(M, I, O, \leq, \mathcal{D})$  **fault model**

- $M \in Sig$ , **reference model** 參照模型
- $\leq \subseteq Sig \times Sig$ , **conformance relation**  
(I/O equivalence or I/O reduction)
- $\mathcal{D} \subseteq Sig$ , **fault domain** 錯誤域

# Test Cases, Test Suite

**Test case** of deterministic *Sig*:

I/O sequence  $\pi = x_1/y_1 \dots x_k/y_k \in \Sigma^*$

- $M$  **passes**  $\pi$ , if  $\pi \in L(M)$
- $M$  **fails**  $\pi$ , if  $\pi \notin L(M)$

**Test suite**  $TS$ : a collection of test cases.

- $M$  **passes**  $TS$ , if  
 $\forall \pi \in TS, M$  passes  $\pi$ .
- $M$  **fails**  $TS$ , if  
 $\exists \pi \in TS, M$  fails  $\pi$ .

# Complete Test Suites

$\mathcal{F}(M, I, O, \leq, \mathcal{D})$ , fault model

**TS**, test suite

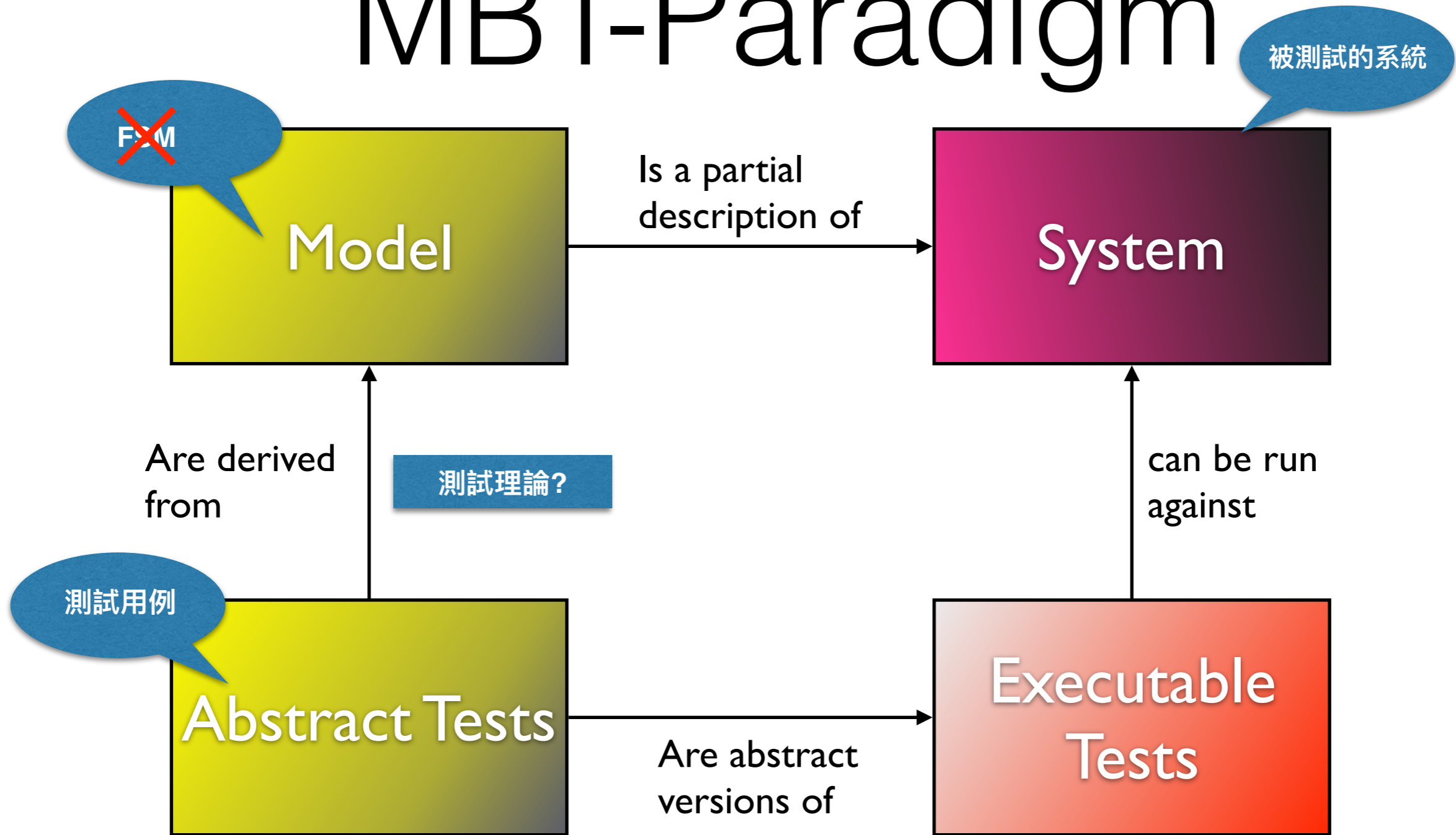
- **Soundness**:  $\forall M' \in \mathcal{D} : M' \leq M \Rightarrow M' \underline{\text{pass}} \mathbf{TS}$
- **Exhaustiveness**:  $\forall M' \in \mathcal{D} : M' \underline{\text{pass}} \mathbf{TS} \Rightarrow M' \leq M$
- **Completeness**: Soundness + Exhaustiveness  
$$\forall M' \in \mathcal{D} : M' \leq M \Leftrightarrow M' \underline{\text{pass}} \mathbf{TS}$$



# Testing Theories

- **Deterministic FSM:**
- **T-Method**  $\mathcal{F}(M, \leq, \mathcal{D}_O)$
- **W-Method, Wp-Method**  $\mathcal{F}(M, \leq, \mathcal{D}_m)$

# MBT-Paradigm



# Motivation

- Many physical systems have **infinite – even uncountable – state spaces**, because they involve real-valued observables like
  - time
  - speed
  - thrust
  - temperature . . .

# Motivation

Which types of system fall into this category?



# Where is our innovation useful?

Airbag controller



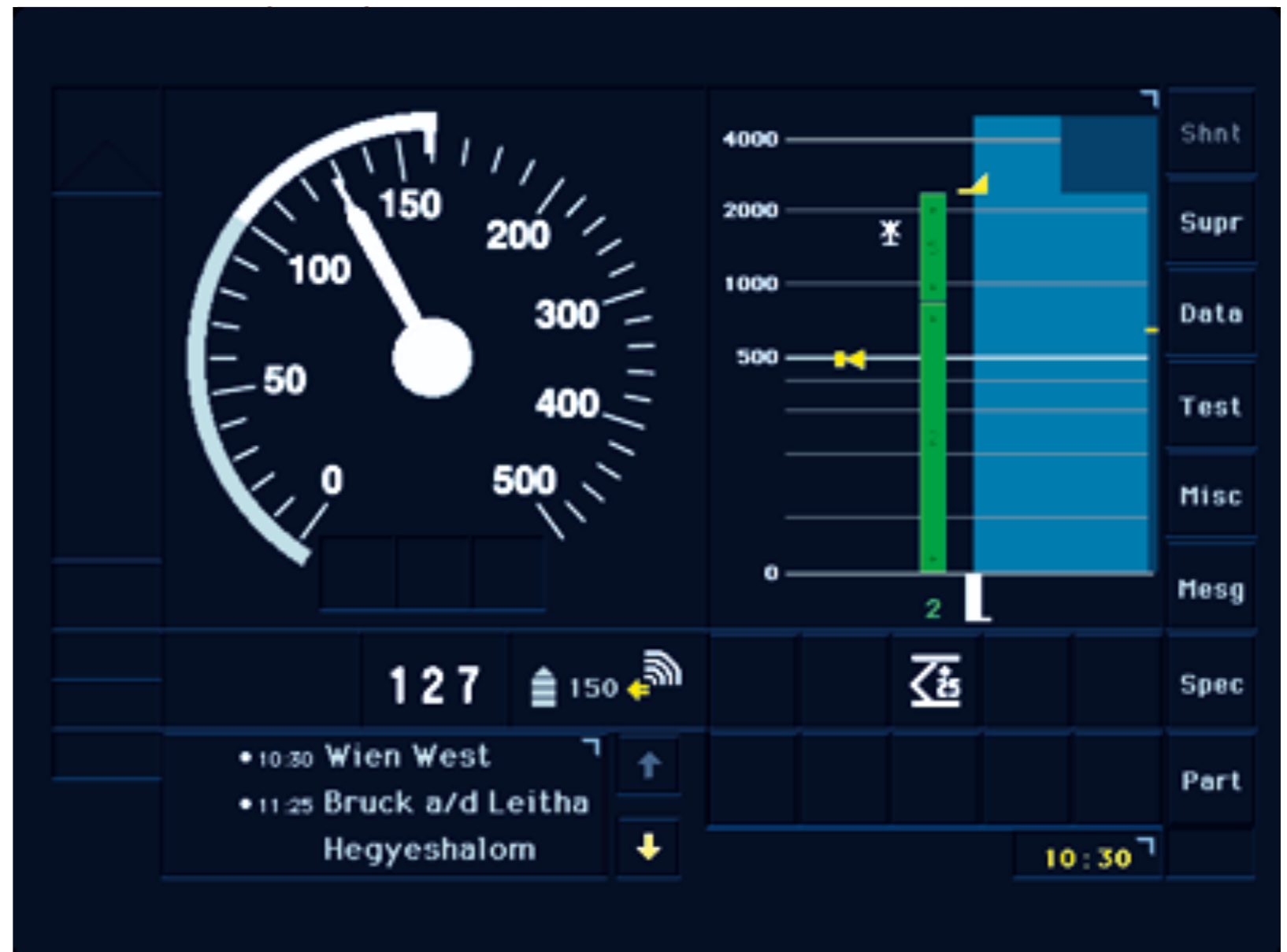
# Where is our innovation useful?

Aircraft thrust reversal



# Where is our innovation useful?

Train speed supervision





# Translation of Testing Theories

HW+SW  $\longrightarrow$

Mathematical models:

- Finite State Machines (FSM)( $Sig_2$ )  
finite directed graphs with labels
- RIOSTS( $Sig_1$ )  
infinite directed graphs

$$Sig_1 \xrightarrow{\text{model map } T} Sig_2$$

$$\begin{array}{ccc} \mathcal{F}(S, \leq_1, \mathcal{D}_1) & \xrightarrow{\text{model map } T} & \mathcal{F}(M, \leq_2, \mathcal{D}_2) \\ \text{complete} \vdots & & \vdots \text{complete} \\ TS_1 & \xleftarrow{\text{test case map } T^*} & TS_2 \end{array}$$

# Satisfaction Condition (1)

$$(T, T^*) : \mathcal{F}(\text{Sig}_1) \times \mathbf{tc}(\text{Sig}_2) \not\rightarrow \mathcal{F}(\text{Sig}_2) \times \mathbf{tc}(\text{Sig}_1)$$

$$\mathcal{F}(S, \leq_1, \mathcal{D}_1) \xrightarrow{T} \mathcal{F}(T(S), \leq_2, \mathcal{D}_2)$$

- $\forall S' \in \mathcal{D}_1, T(S') \in \mathcal{D}_2$
- $S' \leq_1 S \Leftrightarrow T(S') \leq_2 T(S)$

$$\begin{array}{ccc} S & \longrightarrow & T(S) \\ \leq_1 \downarrow & & \downarrow \leq_2 \\ S' & \longrightarrow & T(S') \end{array}$$

# Satisfaction Condition (2)

$$(T, T^*) : \mathcal{F}(\text{Sig}_1) \times \mathbf{tc}(\text{Sig}_2) \not\rightarrow \mathcal{F}(\text{Sig}_2) \times \mathbf{tc}(\text{Sig}_1)$$
$$\mathcal{F}(S, \leq_1, \mathcal{D}_1) \xrightarrow{T} \mathcal{F}(T(S), \leq_2, \mathcal{D}_2)$$

- $\forall S' \in \mathcal{D}_1, U \in \mathbf{tc}(\text{Sig}_2) : T^*(U) \in \mathbf{tc}(\text{Sig}_1)$
- $T(S') \text{ pass}_2 U \Leftrightarrow S' \text{ pass}_1 T^*(U)$

$$\begin{array}{ccc} S' & \longrightarrow & T(S') \\ \text{pass}_1 \downarrow & & \downarrow \text{pass}_2 \\ T^*(U) & \longleftarrow & U \end{array}$$

**Theorem** Let  $(T, T^*)$  satisfy the satisfaction conditions. Then for any **ts** complete test suite of  $\mathcal{F}(T(S), \leq_2, \mathcal{D}_2)$ ,  $T^*(\mathbf{ts})$  is a complete test suite of  $\mathcal{F}(S, \leq_1, \mathcal{D}_1)$ .

$$\begin{aligned}
 \mathcal{S}' \leq_1 \mathcal{S} &\Leftrightarrow T(\mathcal{S}') \leq_2 T(\mathcal{S}) && [\text{satisfaction condition (1)}] \\
 &\Leftrightarrow \forall U \in \mathbf{ts}, T(\mathcal{S}') \underline{\text{pass}}_2 U && [\mathbf{ts} \text{ is a complete test suite of } \mathcal{F}(T(S), \leq_2, \mathcal{D}_2)] \\
 &\Leftrightarrow \forall U \in \mathbf{ts}, \mathcal{S}' \underline{\text{pass}}_1 T^*(U) && [\text{satisfaction condition (2)}]
 \end{aligned}$$

# Testing Theories of Finite State Machines (*Sig*<sub>2</sub>) — Recall

# Finite State Machines

$$M = (Q, q_0, I, O, h)$$

- $Q \neq \emptyset$ : finite set of states 狀態集
- $q_0 \in Q$ : initial state 初始狀態
- $I \neq \emptyset$ : finite set of input alphabet 輸入字母表
- $O \neq \emptyset$ : finite set of output alphabet 輸出字母表
- $h \subseteq Q \times I \times O \times Q$ : transition relation 狀態遷移關係

# Language of FSM

$$M = (Q, q_0, I, O, h)$$

- $L(q) := \{\pi \in \Sigma^* \mid \exists q' \in Q, q \xrightarrow{\pi} q'\}$  **language of  $q$**
- $q \sim q' :\Leftrightarrow L(q) = L(q')$   **$q$   $q'$  are equivalent**
- $L(M) := L(q_0)$  **language of  $M$**



# Conformance Relations

$M = (Q, q_0, I, O, h)$ ,  $M' = (Q', q'_0, I, O, h')$  two FSM.

- $M$  and  $M'$  are **I/O equivalent** :  $L(M') = L(M)$
- $M'$  is an **I/O reduction** of  $M$  :  $L(M') \subseteq L(M)$

# Testing Theories

- **Deterministic FSM:**
- **T-Method**  $\mathcal{F}(M, \leq, \mathcal{D}_O)$
- **W-Method, Wp-Method**  $\mathcal{F}(M, \leq, \mathcal{D}_m)$

# State Cover

**State cover**  $V$  of  $M = (Q, q_0, I, O, h)$

- $V \subseteq L(M)$

- $\varepsilon \in V$

- $\forall q \in Q, \exists \pi \in V : q_0 \xrightarrow{\pi} q.$

$q_0\text{-after-}\pi = q$

# Transition Cover

## Concatenation

For any  $A, B \neq \emptyset \subseteq \Sigma^*$ .

$$A.B := \{\pi.\iota \mid \pi \in A, \iota \in B\}$$

Example:  $\Sigma = \{a, b, c\}$

$$A = \{\varepsilon\}, B = \{a.b\}, C = \{a, c\}$$

$$A.B = \{\varepsilon.a.b\} = \{a.b\} = B$$

$$B.C = \{a.b.a, a.b.c\}$$

**Transition cover**  $P$  of  $M = (Q, q_0, I, O, h)$

- $P \subset L(M)$
- $\varepsilon \in P$
- $\forall q \in Q, \sigma = x/y \in L(q), \exists \pi \in P : q_0 \xrightarrow{\pi} q \wedge \pi.\sigma \in P$

$V$  is a state cover

$$\begin{aligned} \Rightarrow V \oplus_M (\{\varepsilon\} \cup \Sigma) &= (V.(\{\varepsilon\} \cup \Sigma)) \cap L(M) \\ &= V \cup \{\pi.\sigma \in L(M) \mid \pi \in V, \sigma \in \Sigma\} \end{aligned}$$

is a transition cover

# W-Method

M. P. Vasilevskii 1973 and Tsun S. Chow 1978

$\mathcal{F}(M, I, O, \leq, \mathcal{D}_m)$ , fault model

$$\mathcal{D}_m = \{M' = (Q', q'_0, I, O, h') \mid |Q'| \leq m\}$$

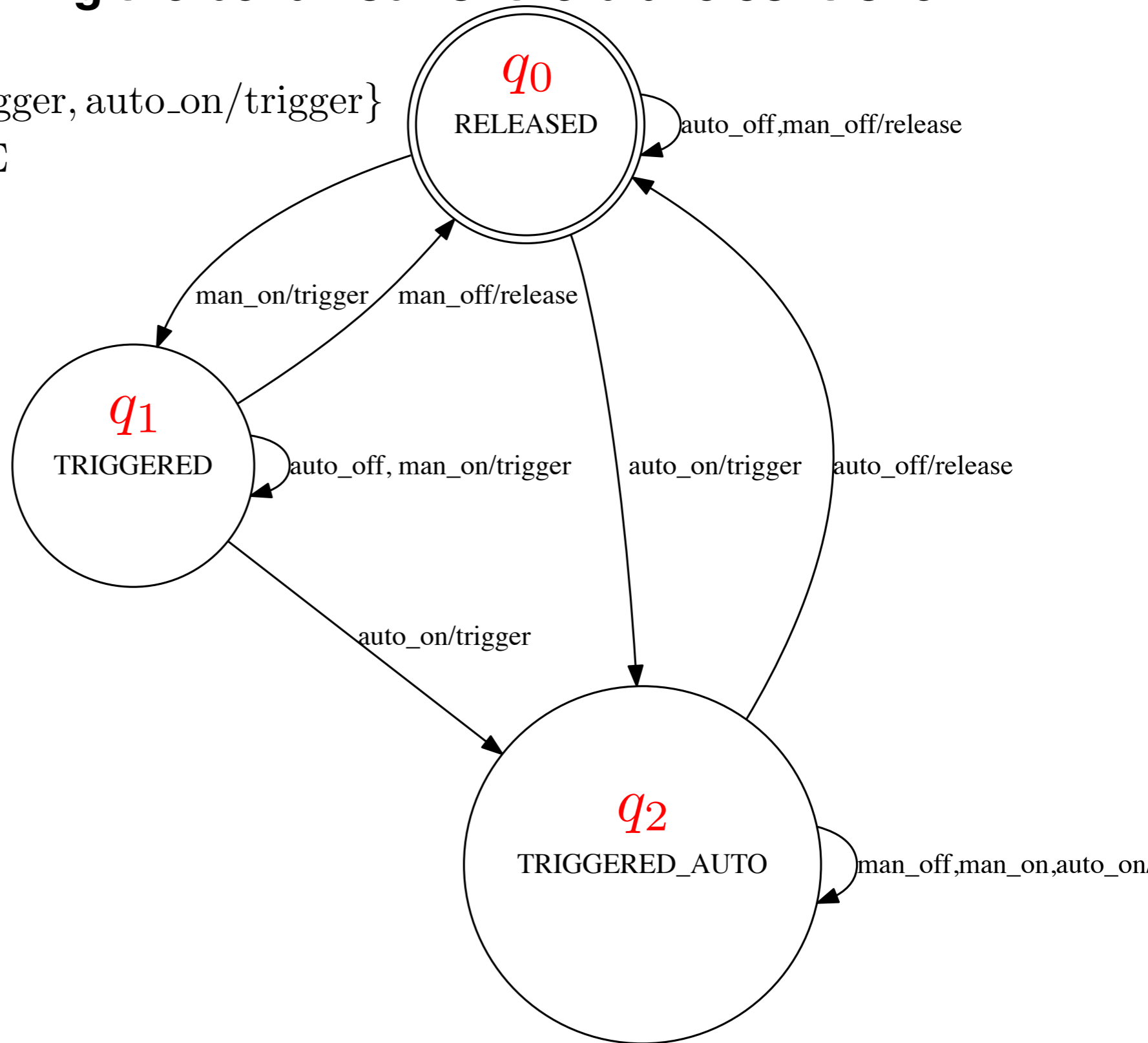
# Characterization Set

**Characterization set**  $W$  of  $M = (Q, q_0, I, O, h)$

- $W \subseteq \Sigma^*$  is a set of I/O sequences
- $\forall q_1 \neq q_2 \in Q, \exists \tau_1 \neq \tau_2 \in W : \tau_{1_I} = \tau_{2_I} \wedge \tau_i \in L(q_i), i = 1, 2$

# Finite State Machine modelling the behaviour of the brake controller

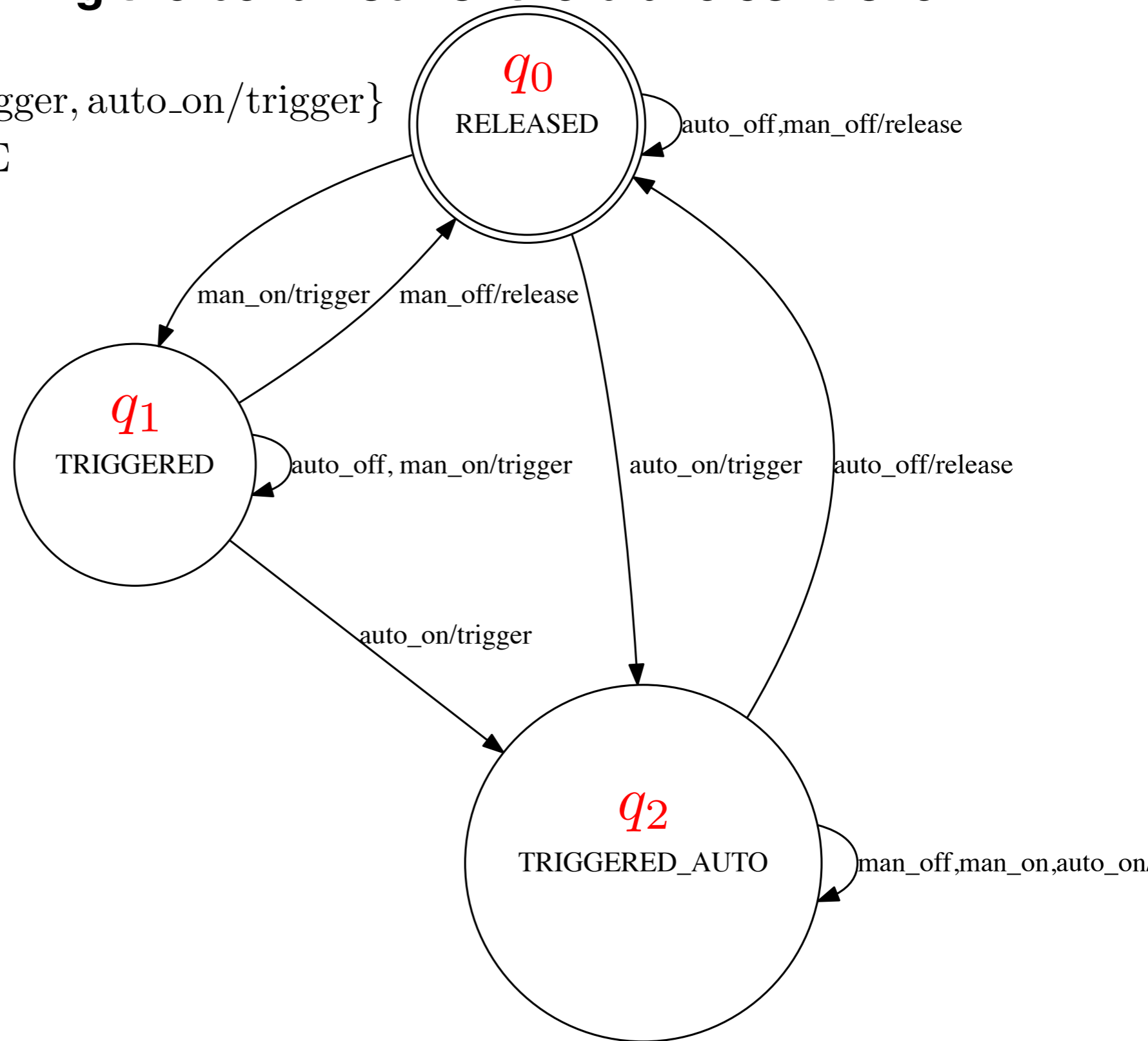
- state cover  $V = \{\varepsilon, \text{man\_on}/\text{trigger}, \text{auto\_on}/\text{trigger}\}$
- transition cover  $P = V \cup V \oplus \Sigma$



# Finite State Machine modelling the behaviour of the brake controller

- state cover  $V = \{\varepsilon, \text{man\_on}/\text{trigger}, \text{auto\_on}/\text{trigger}\}$
- transition cover  $P = V \cup V \oplus \Sigma$

$q_0 \xrightarrow{\text{auto\_off}/\text{release}} q_0$   
 $q_1 \xrightarrow{\text{auto\_off}/\text{trigger}} q_1$   
 $q_2 \xrightarrow{\text{auto\_off}/\text{release}} q_0$   
 $q_0 \xrightarrow{\text{man\_off}/\text{release}} q_0$   
 $q_2 \xrightarrow{\text{man\_off}/\text{trigger}} q_2$

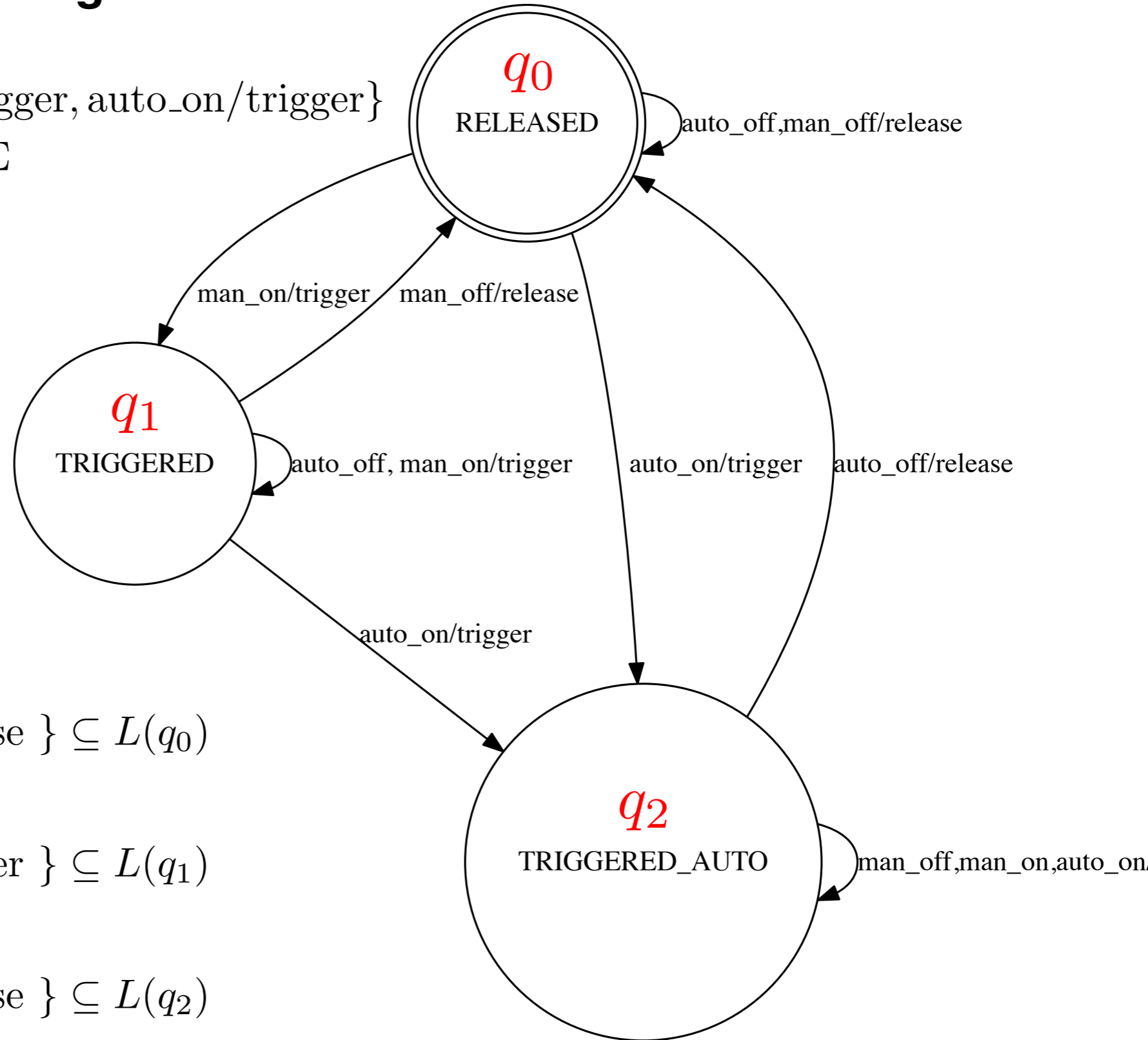




# Finite State Machine modelling the behaviour of the brake controller

- state cover  $V = \{\varepsilon, \text{man\_on}/\text{trigger}, \text{auto\_on}/\text{trigger}\}$
- transition cover  $P = V \cup V \oplus \Sigma$

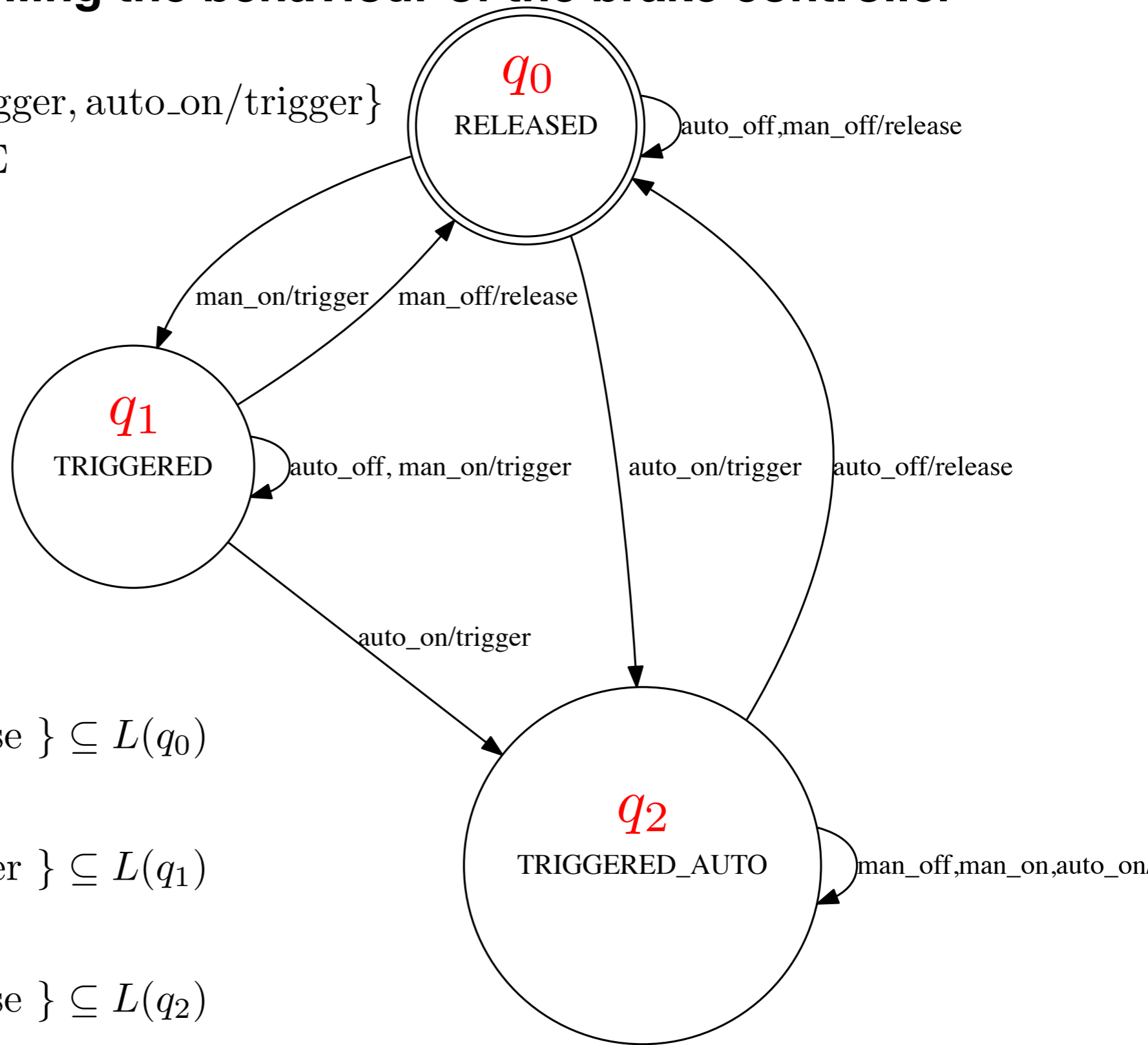
$q_0 \xrightarrow{\text{auto\_off}/\text{release}} q_0$   
 $q_1 \xrightarrow{\text{auto\_off}/\text{trigger}} q_1$   
 $q_2 \xrightarrow{\text{auto\_off}/\text{release}} q_0$   
 $q_0 \xrightarrow{\text{man\_off}/\text{release}} q_0$   
 $q_2 \xrightarrow{\text{man\_off}/\text{trigger}} q_2$



- $\{\text{man\_off}/\text{release}, \text{auto\_off}/\text{release}\} \subseteq L(q_0)$
- $\{\text{man\_off}/\text{release}, \text{auto\_off}/\text{trigger}\} \subseteq L(q_1)$
- $\{\text{man\_off}/\text{trigger}, \text{auto\_off}/\text{release}\} \subseteq L(q_2)$

# Finite State Machine modelling the behaviour of the brake controller

- state cover  $V = \{\varepsilon, \text{man\_on}/\text{trigger}, \text{auto\_on}/\text{trigger}\}$
- transition cover  $P = V \cup V \oplus \Sigma$



- $\{\text{man\_off}/\text{release}, \text{auto\_off}/\text{release}\} \subseteq L(q_0)$

- $\{\text{man\_off}/\text{release}, \text{auto\_off}/\text{trigger}\} \subseteq L(q_1)$

- $\{\text{man\_off}/\text{trigger}, \text{auto\_off}/\text{release}\} \subseteq L(q_2)$

- characterization set  $W = \{\text{man\_off}/\text{trigger}, \text{man\_off}/\text{release}, \text{auto\_off}/\text{trigger}, \text{auto\_off}/\text{release}\}$

# W-Method

Every **TS** =  $V \oplus \left( \bigcup_{i=0}^{m-n+1} \Sigma^i \right) \oplus W$   
is a complete test suite of  $\mathcal{F}(M, I, O, \sim, \mathcal{D}_m)$ ,  $n = |Q|$

# W-Method

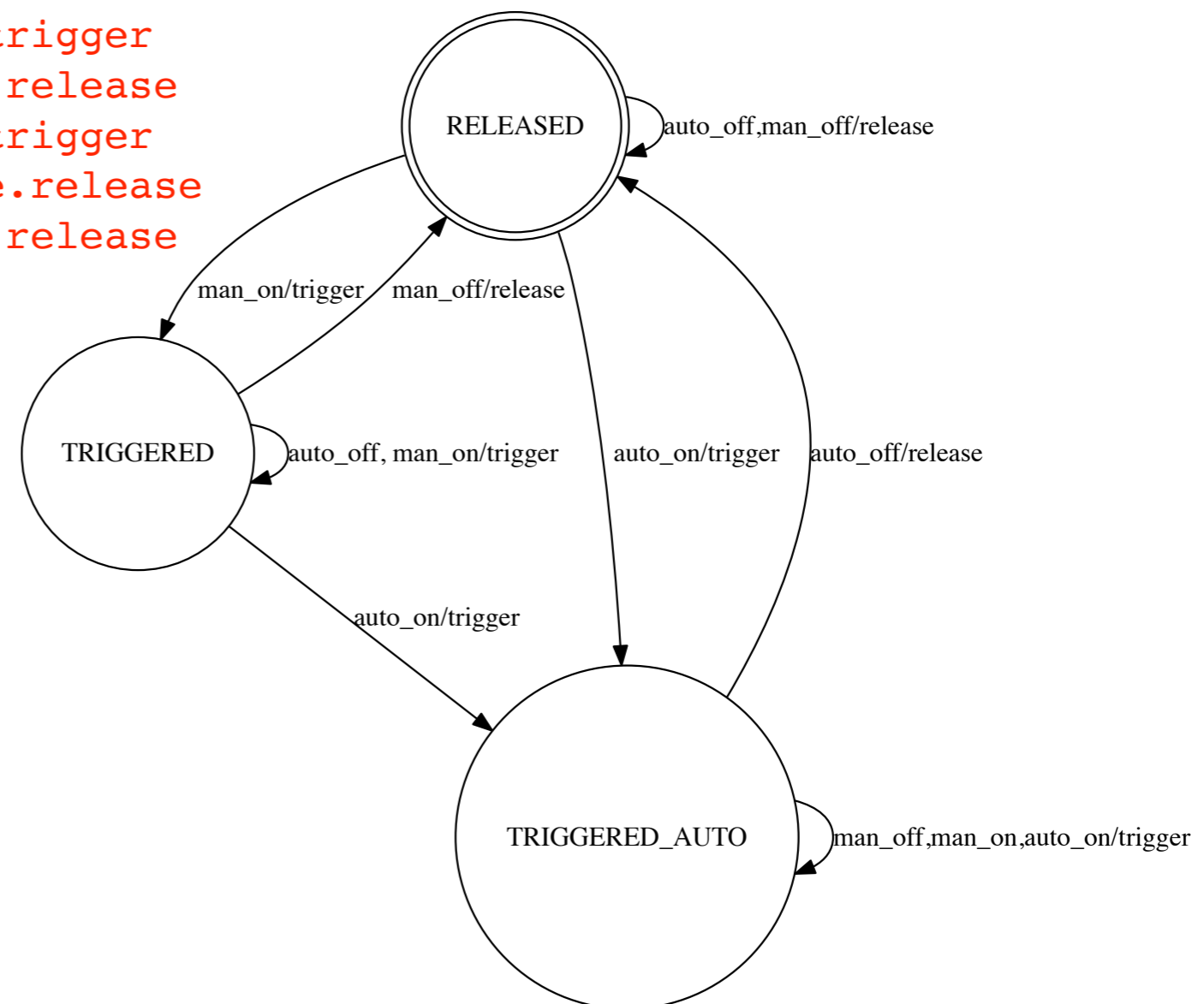
Every  $\mathbf{TS} = V \oplus \left( \bigcup_{i=0}^{m-n+1} \Sigma^i \right) \oplus W$   
is a complete test suite of  $\mathcal{F}(M, I, O, \sim, \mathcal{D}_m)$ ,  $n = |Q|$

$$\mathbf{TS}_I = V_I \cdot \left( \bigcup_{i=0}^{m-n+1} I^i \right) \cdot W_I$$

# Test cases for hypothesis $m = 3$

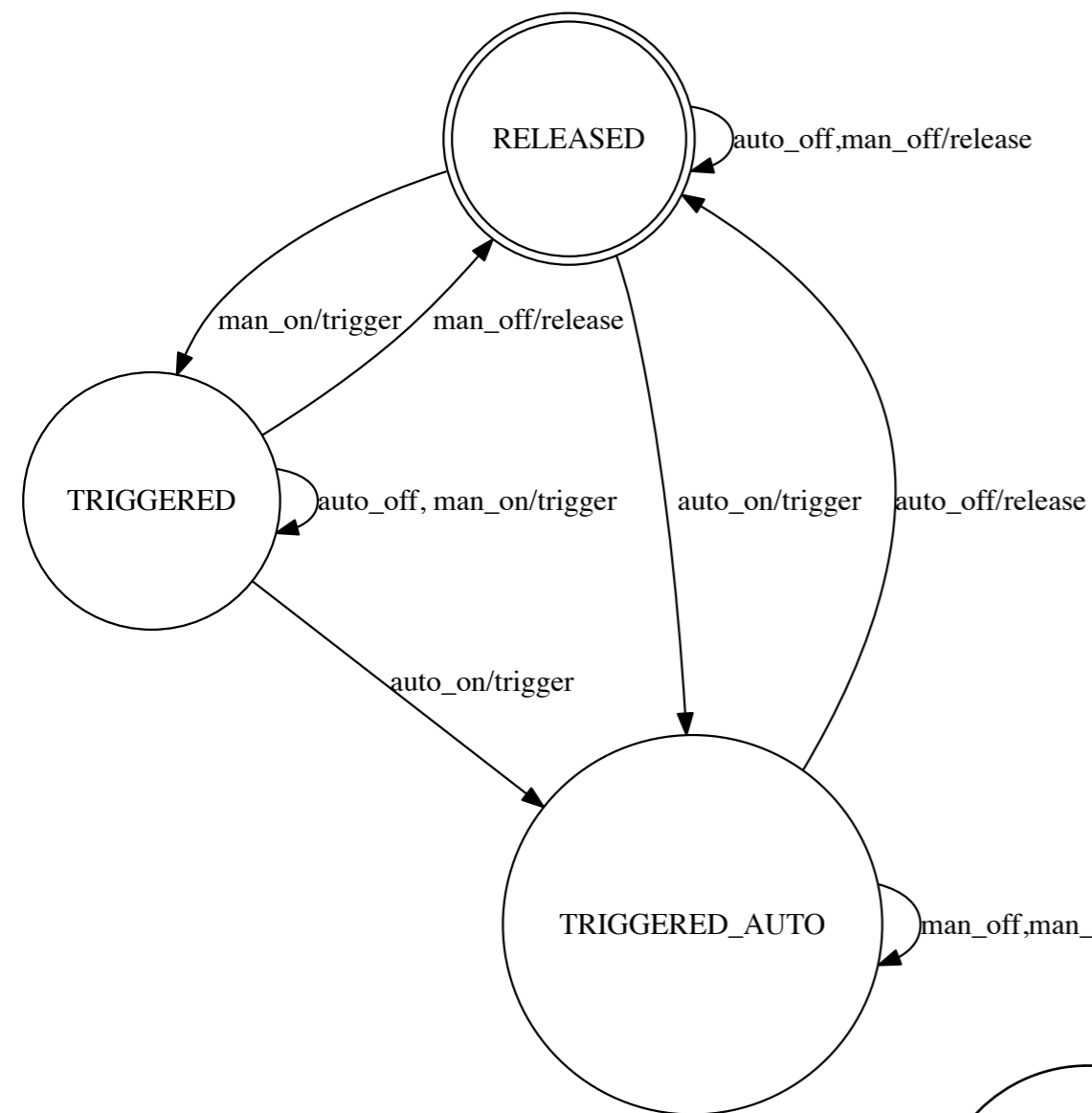
This test suite uncovers every error, provided that the implementation has at most 3 states

1. man\_on.man\_on.auto\_off/trigger.trigger.trigger
2. man\_on.man\_on.man\_off/trigger.trigger.release
3. man\_on.auto\_on.auto\_off/trigger.trigger.release
4. man\_on.auto\_on.man\_off/trigger.trigger.trigger
5. man\_on.man\_off.auto\_off/trigger.release.release
6. man\_on.man\_off.man\_off/trigger.release.release
7. man\_on.auto\_off.auto\_off/trigger.trigger.trigger
8. man\_on.auto\_off.man\_off/trigger.trigger.release
9. auto\_on.man\_on.auto\_off/trigger.trigger.release
10. auto\_on.man\_on.man\_off/trigger.trigger.trigger
11. auto\_on.auto\_on.auto\_off/trigger.trigger.release
12. auto\_on.auto\_on.man\_off/trigger.trigger.trigger
13. auto\_on.man\_off.auto\_off/trigger.trigger.release
14. auto\_on.man\_off.man\_off/trigger.trigger.trigger
15. auto\_on.auto\_off.auto\_off/trigger.release.release
16. auto\_on.auto\_off.man\_off/trigger.release.release
17. man\_off.auto\_off/release.release
18. man\_off.man\_off/release.release
19. auto\_off.auto\_off/release.release
20. auto\_off.man\_off/release.release

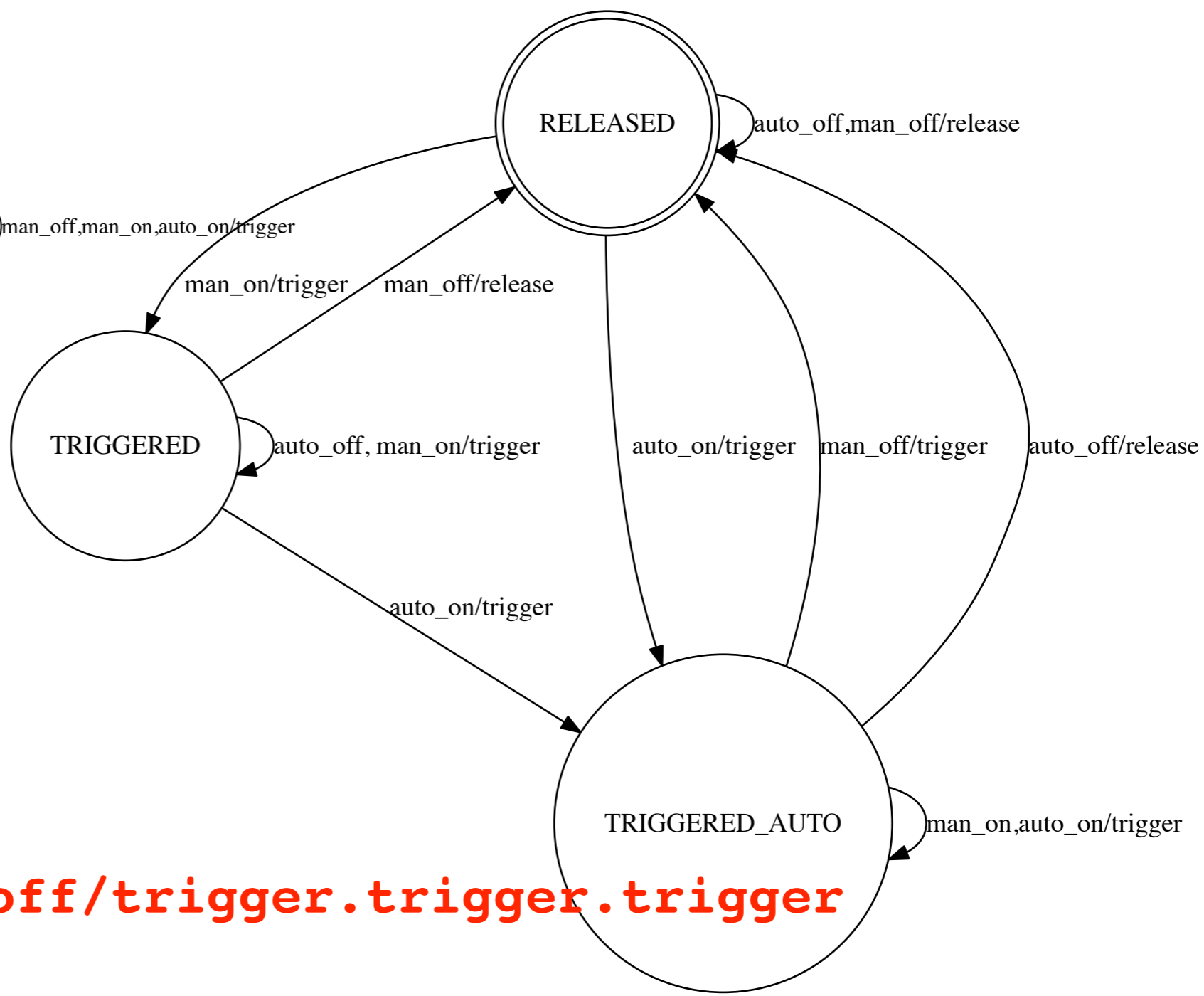


21. man\_off/release is a prefix of 17.
22. auto\_off/release is a prefix of 19.

# Reference model



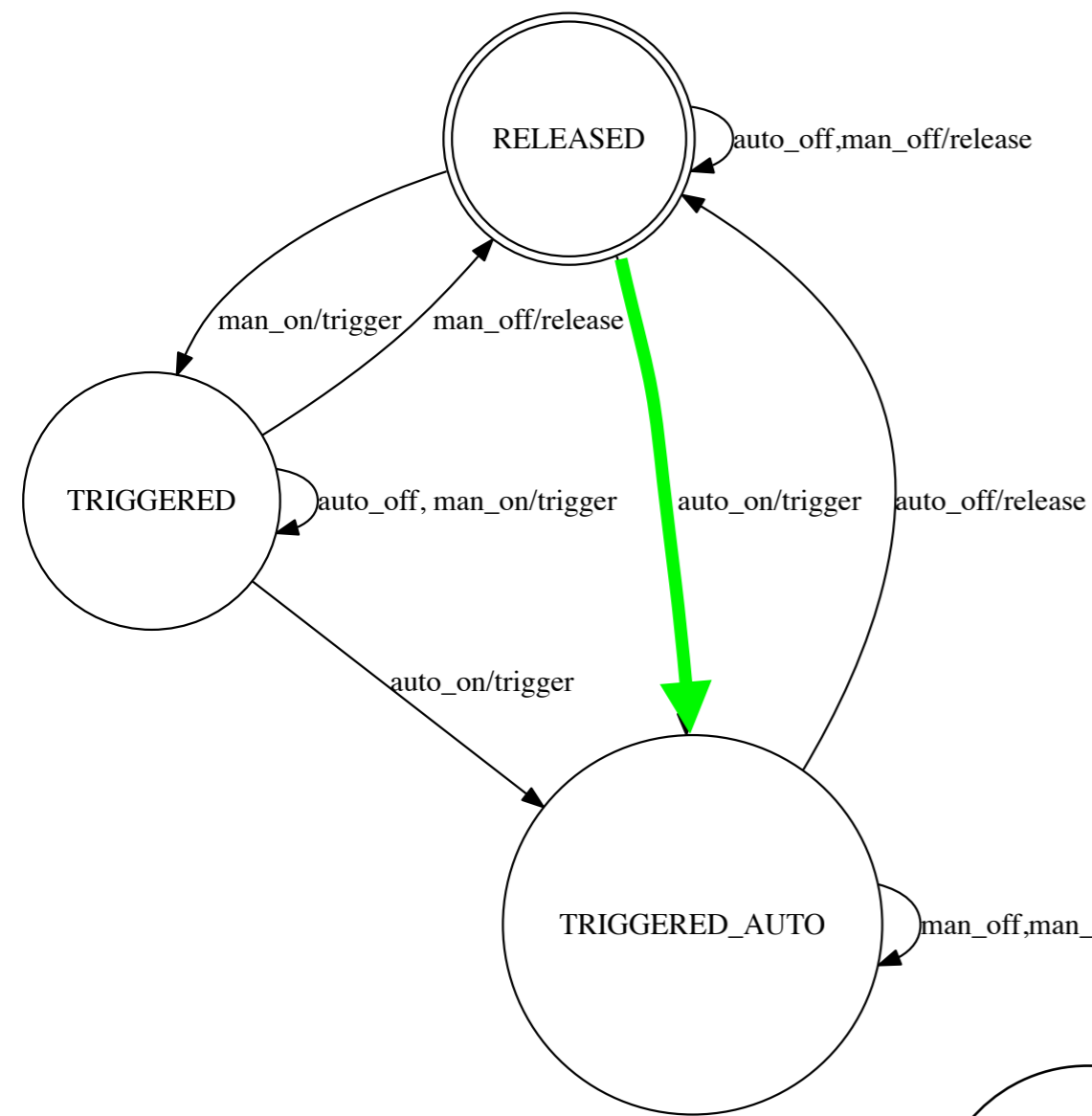
# Faulty implementation



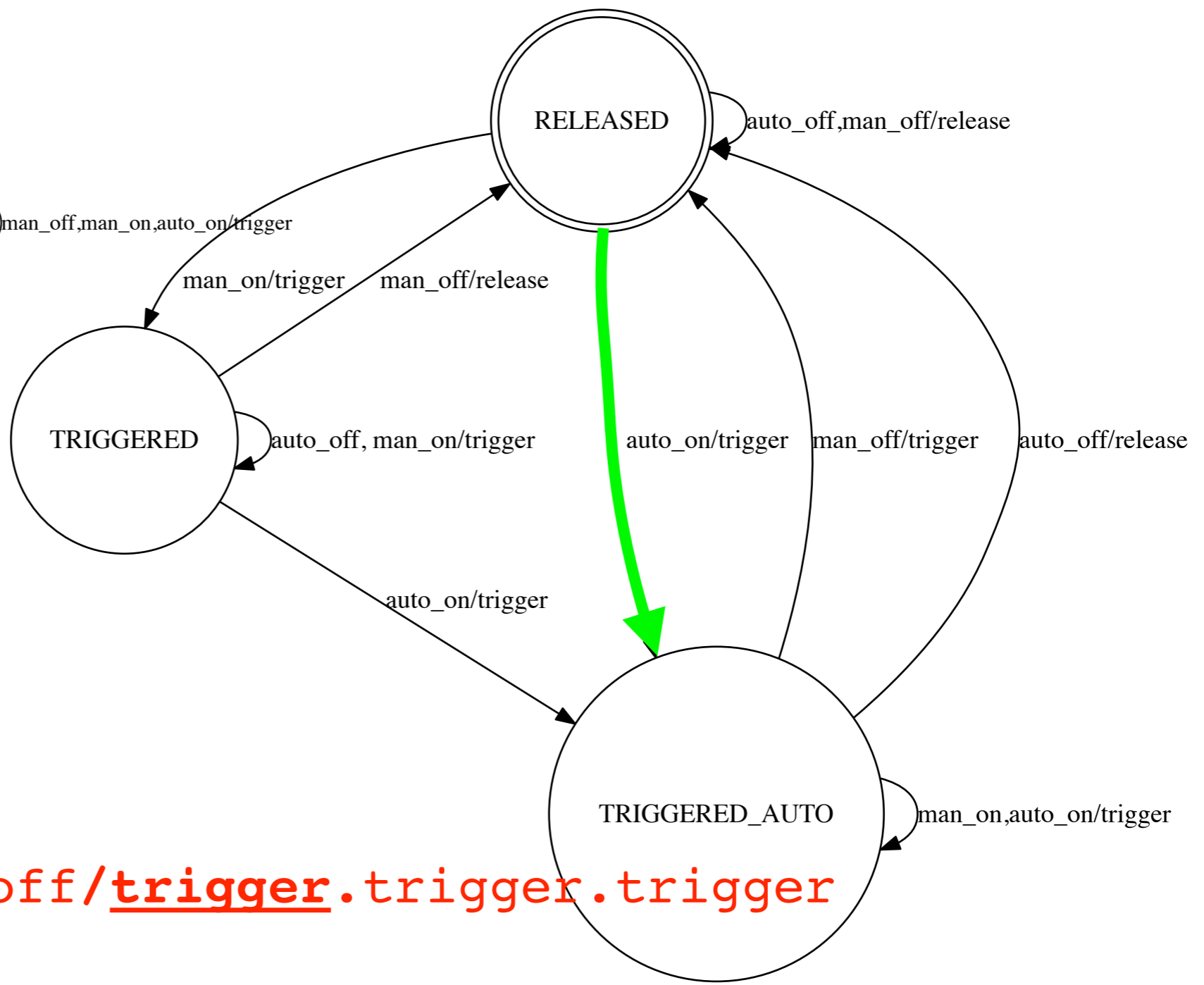
**Test case 14.**

**auto\_on.man\_off.man\_off/trigger.trigger.trigger**

# Reference model



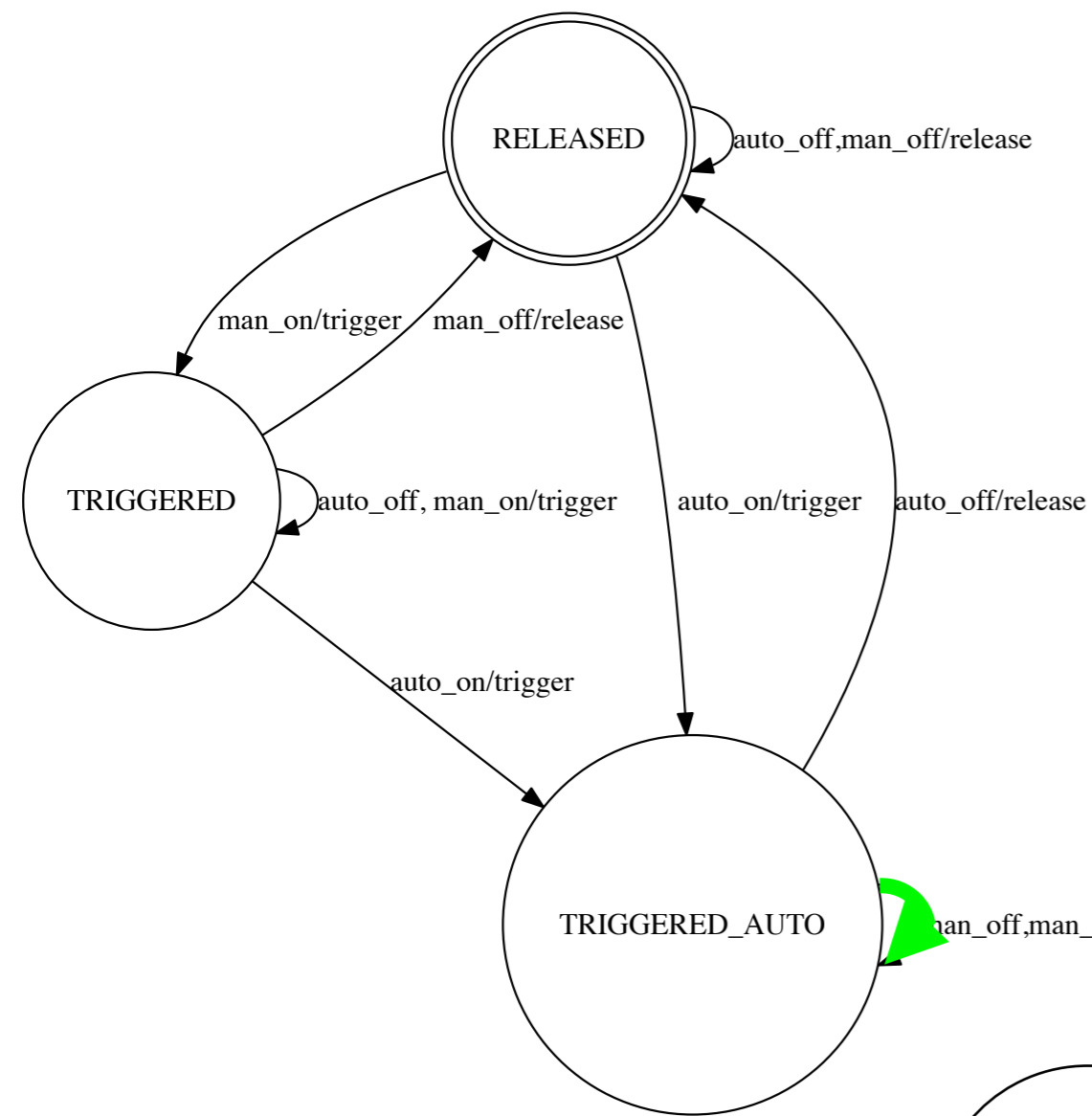
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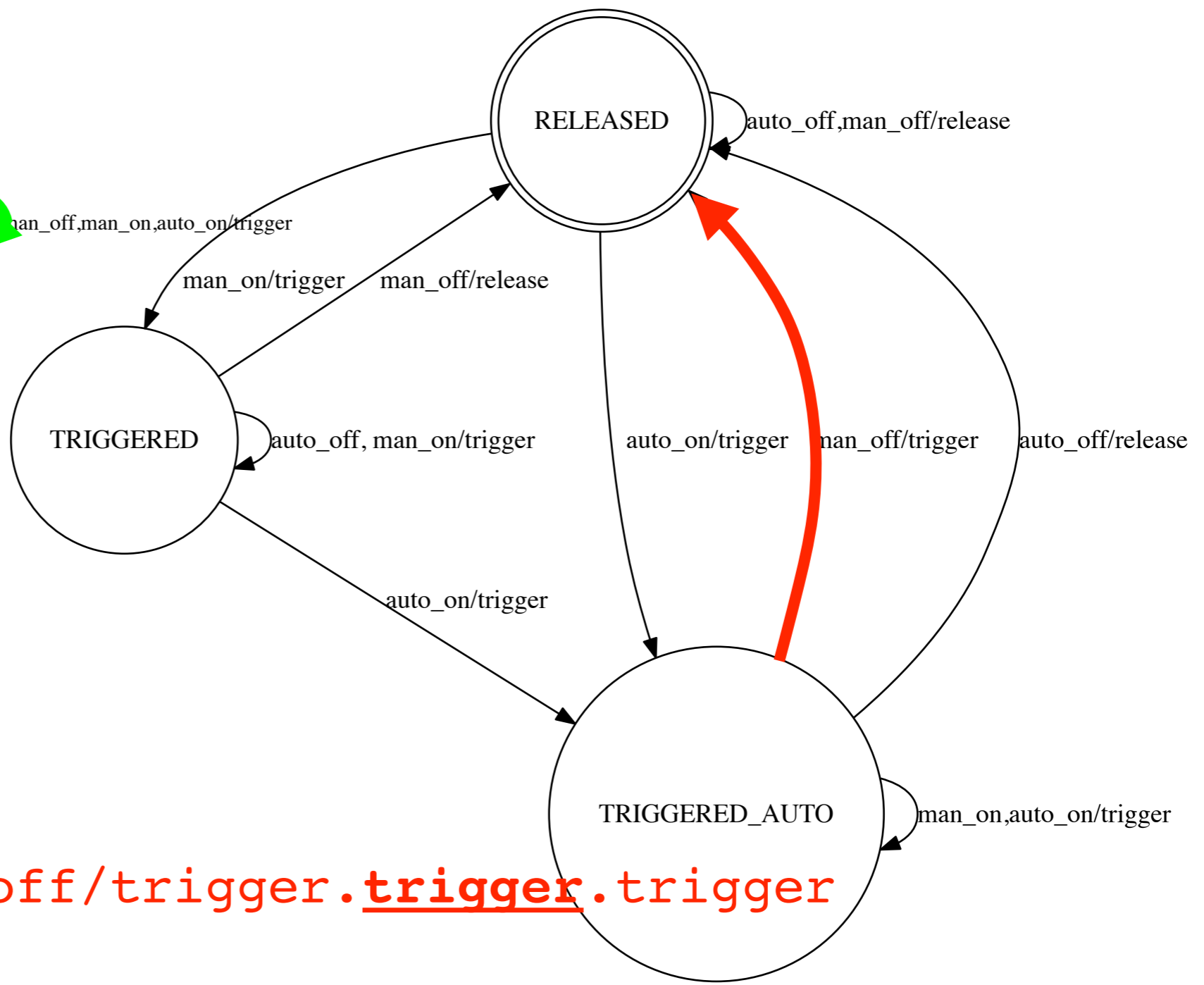
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# Reference model



# Faulty implementation

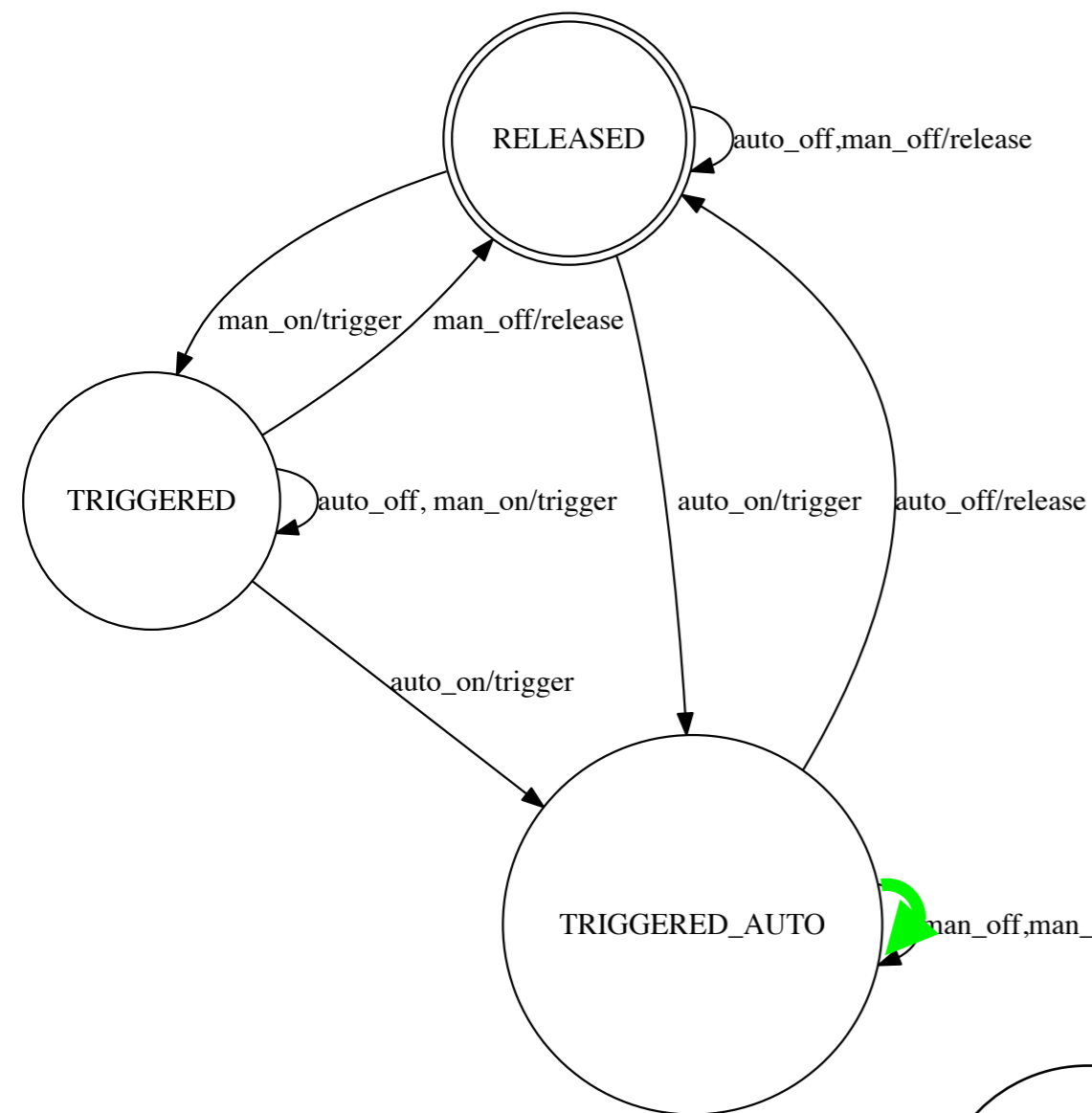


**Test case 14.**

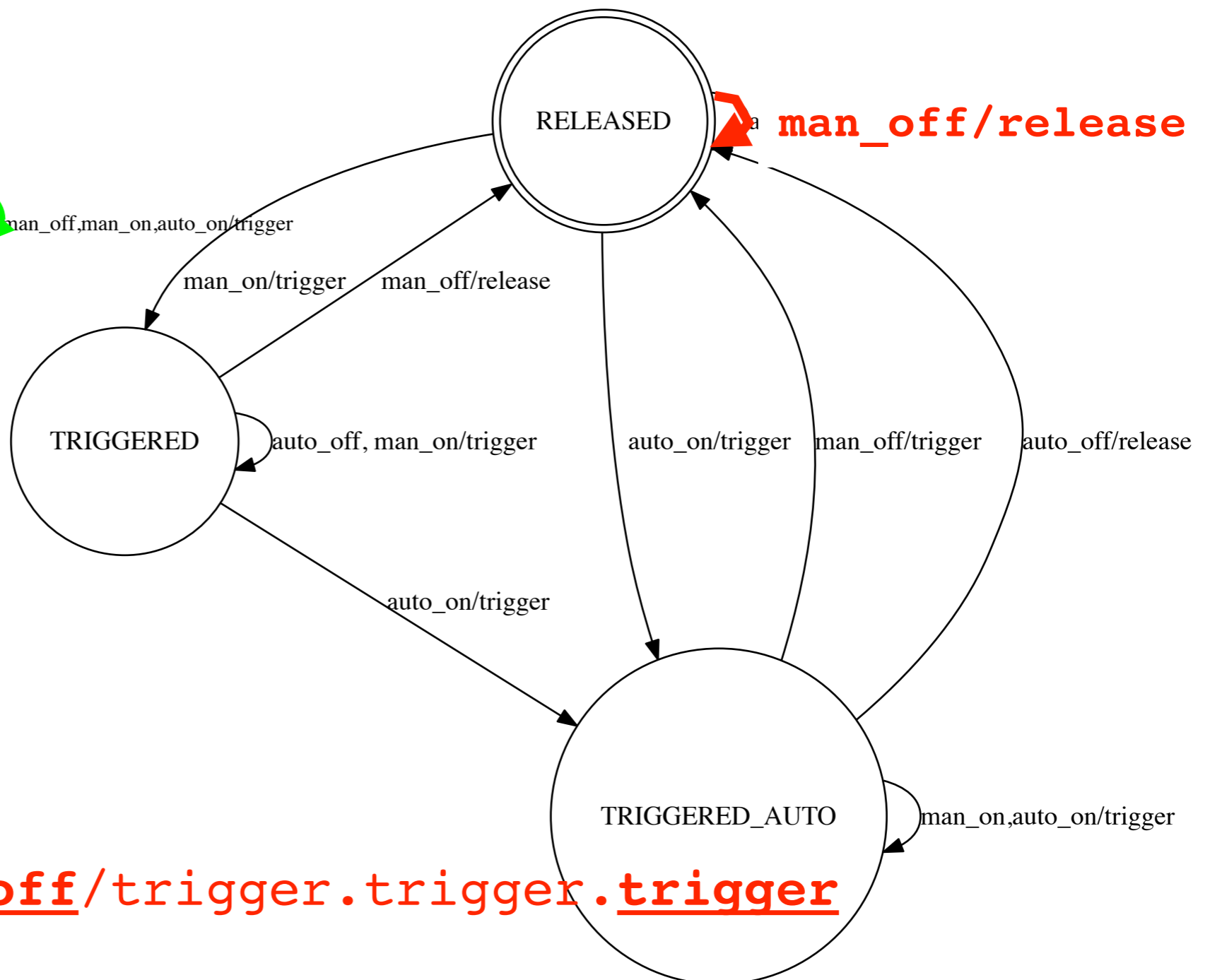
**auto\_on.man\_off.man\_off/trigger.trigger.trigger**



# Reference model



# Faulty implementation



**Test case 14.**

**auto\_on.man\_off.man\_off/trigger.trigger.trigger**

# Wp-Method

$\mathcal{F}(M, I, O, \leq, \mathcal{D}_m)$ , fault model

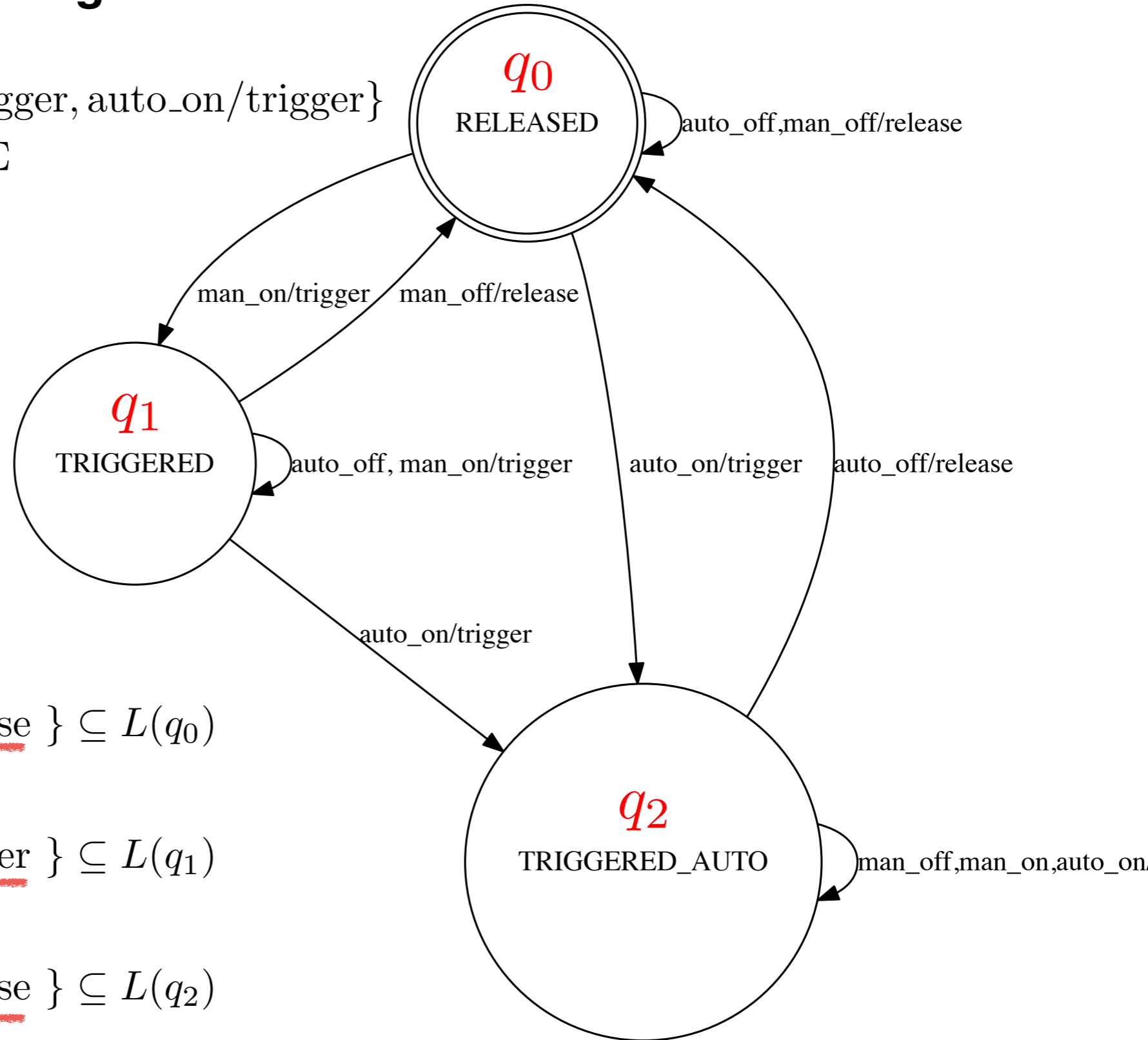
$$\mathcal{D}_m = \{M' = (Q', q'_0, I, O, h') \in Sig_1 \mid |Q'| \leq m\}$$

1.  $V$  a state cover of  $M$ .
2.  $P = V \oplus (\{\varepsilon\} \cup \Sigma)$  a transition cover of  $M$
3.  $R = P \setminus V$ .
4.  $W$  a characterisation set of  $M$ .
5.  $\{W_0, \dots, W_{n-1}\}$  *state identification sets* of  $M$ , such that
  - $W_i \subseteq \text{pref}(W)$  for  $i = 0, \dots, n - 1$ .
  - $W_i$  distinguishes  $q_i$  from all other states in  $Q$ .

# Finite State Machine modelling the behaviour of the brake controller

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 $q_2 \xrightarrow{\text{auto\_off}/\text{release}} q_0$   
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 $q_2 \xrightarrow{\text{man\_off}/\text{trigger}} q_2$



- $\{\text{man\_off}/\text{release}, \text{auto\_off}/\text{release}\} \subseteq L(q_0)$   
 $W_0$

- $\{\text{man\_off}/\text{release}, \text{auto\_off}/\text{trigger}\} \subseteq L(q_1)$   
 $W_1$

- $\{\text{man\_off}/\text{trigger}, \text{auto\_off}/\text{release}\} \subseteq L(q_2)$   
 $W_2$

- characterization set  $W = \{\text{man\_off}/\text{trigger}, \text{man\_off}/\text{release}, \text{auto\_off}/\text{trigger}, \text{auto\_off}/\text{release}\}$

# Wp-Method

- $Wp_1 = V \oplus \left( \bigcup_{i=0}^{m-n} \Sigma^i \right) \oplus W$
- $Wp_2 = R \oplus \Sigma^{m-n} \oplus \{W_0, \dots, W_{n-1}\}$
- **TS** =  $Wp_1 \cup Wp_2$   
is a complete test suite of  $\mathcal{F} = (M, I, O, \sim, \mathcal{D}_m)$ .

$$U \oplus \{W_0, \dots, W_{n-1}\} =$$

$$\bigcup_{\pi \in U \wedge q_i = q_0 \text{-after-}\pi} \{\pi\} \cdot W_i$$

# Wp-Method

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# Reactive I/O Transition System ( $Sig_1$ )

# Reactive I/O Transition System

$\mathcal{S} = (S, s_0, R)$  reactive I/O transition system:

- $S = I \times M \times O$ : state space  $S = \{(x, m, y) \mid x \in I, m \in M, y \in O\}$
- $s_0 \in S$ : initial state
- $I \neq \emptyset$ : input alphabet
- $M \neq \emptyset$ : internal state values
- $O \neq \emptyset$ : output alphabet
- $R \subseteq S \times S$ : transition relation



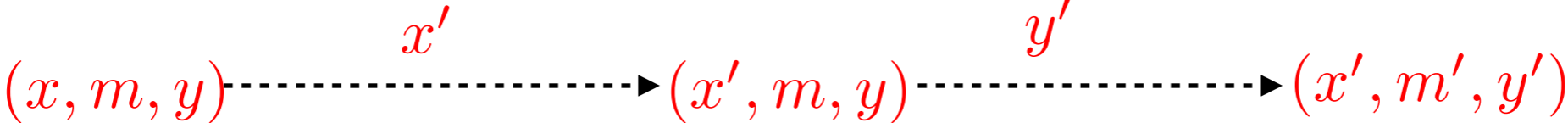
$$S = S_Q \cup S_T, S_Q \cap S_T = \emptyset$$

- $S_Q$ : quiescent states      靜態狀態
- $S_T$ : transient states      瞬變狀態

$$R \subseteq S \times S:$$

- $s_1$  quiescent:  $(s_1, s_2) \in R \Leftrightarrow s_2(m, y) = s_1(m, y)$
- $s_1$  transient :  $(s_1, s_2) \in R \Rightarrow s_2(x) = s_1(x) \wedge s_2 \in S_Q$

$R$  .....▶

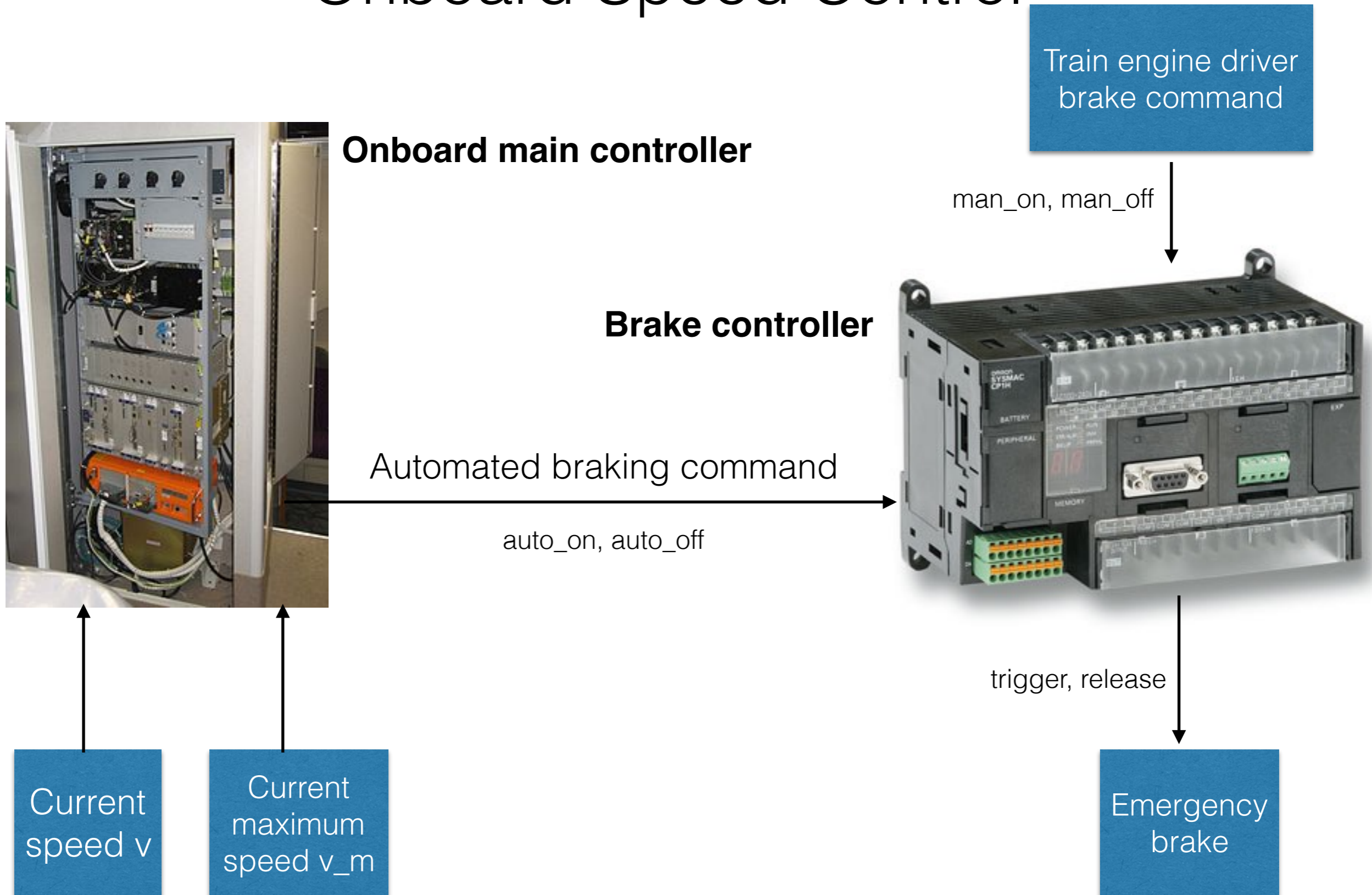


quiescent state

transient state

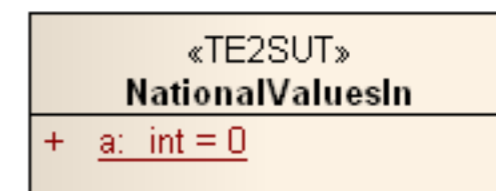
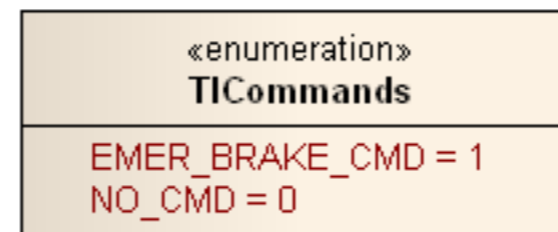
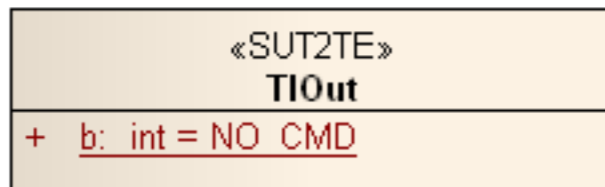
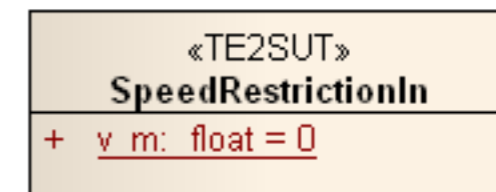
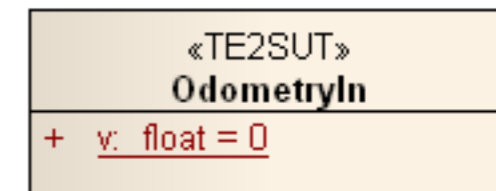
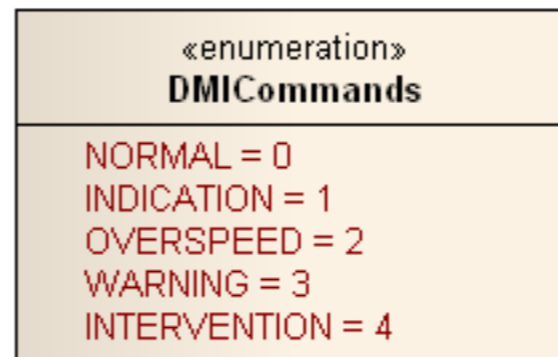
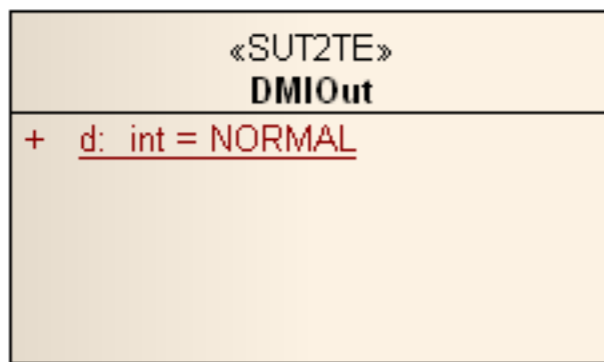
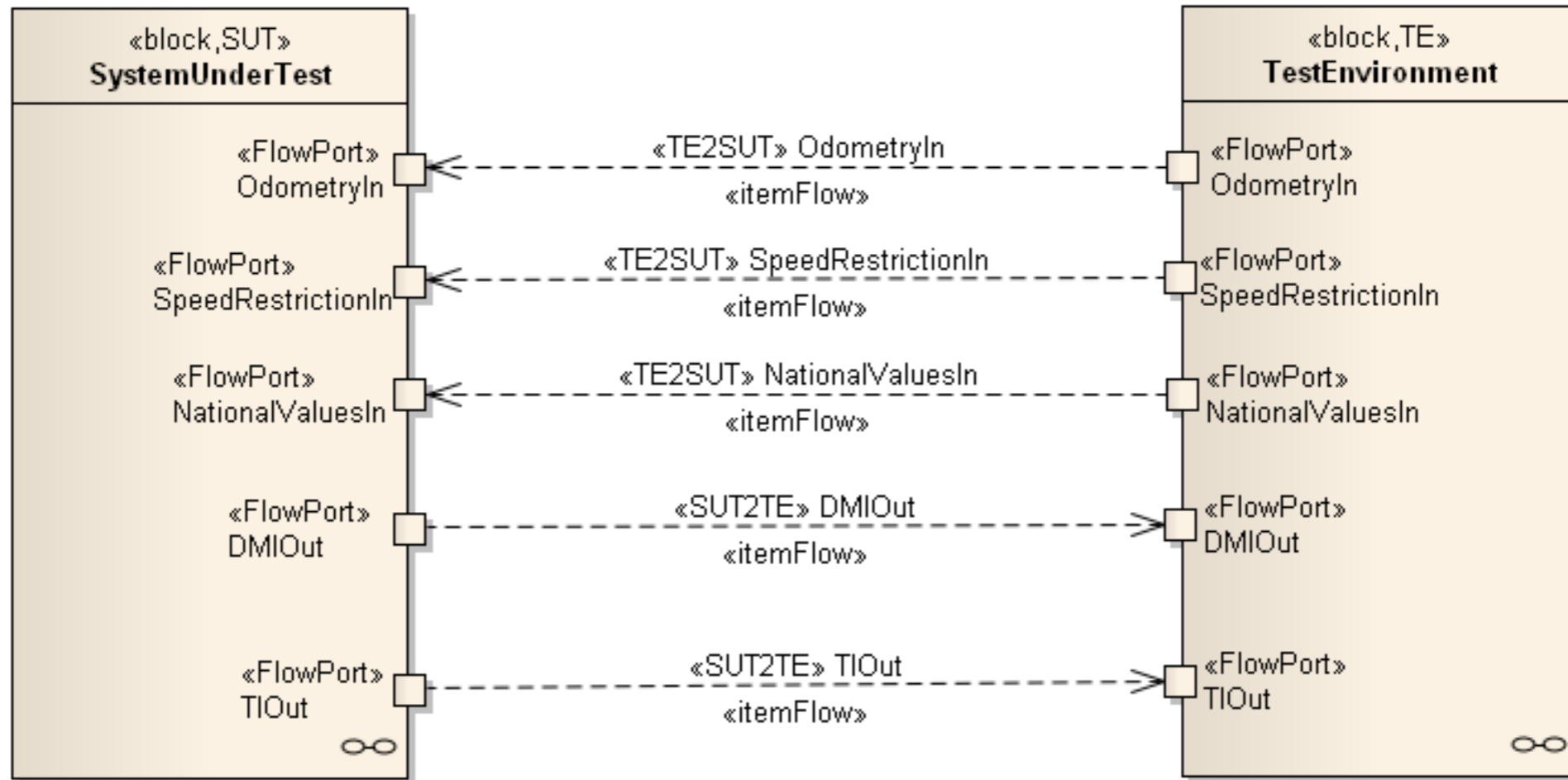
quiescent state

# Recall: Application Scenario – Train Onboard Speed Control



# 最高速限監測器

composite structure SYSTEM



# 最高速限監測器

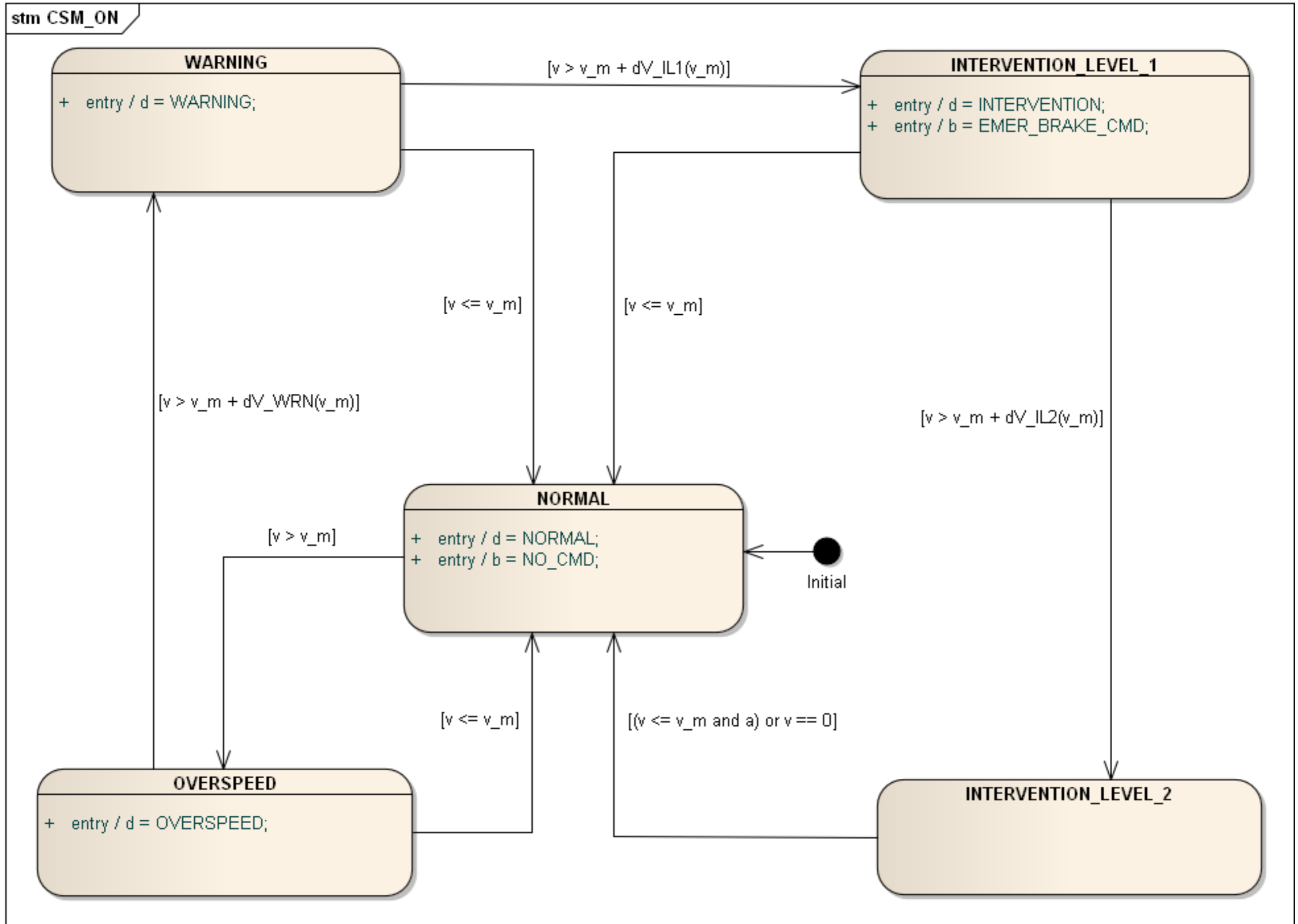
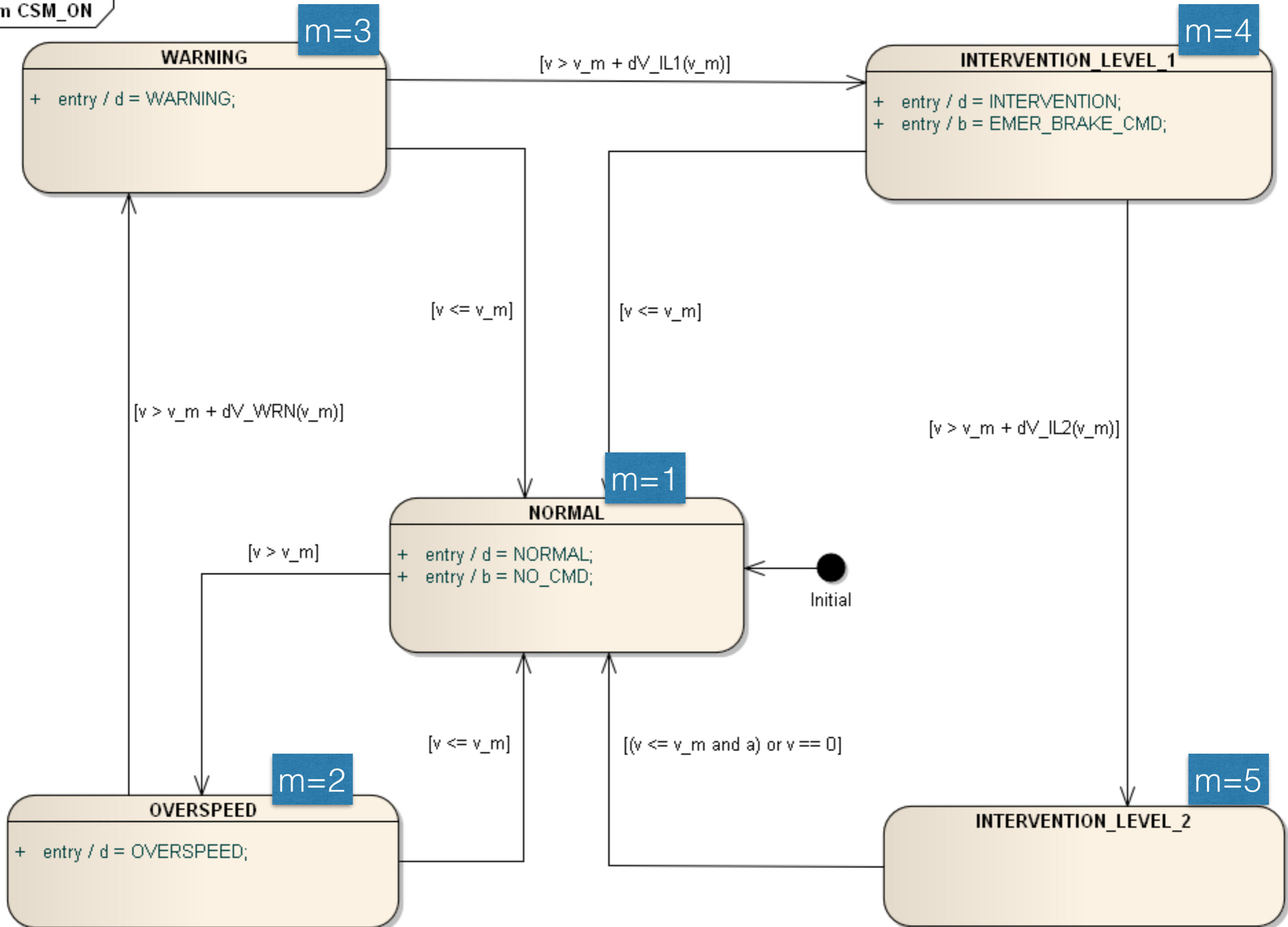


Table 1: Identification of basic states in machine CSM\_ON

<b>State Machine in Basic State</b>	<i>d</i>	<i>m</i>
NORMAL	0	1
OVERSPEED	2	2
WARNING	3	3
INTERVENTION_LEVEL_1	4	4
INTERVENTION_LEVEL_2	4	5

stm CSM\_ON



- $x = (v, v_m, a) \in I, I = [0, 350] \times [0, 350] \times \{0, 1\}$
- $m \in \{1, 2, 3, 4, 5\}$
- $y = (d, b) \in O, O = \{0, 2, 3, 4\} \times \{0, 1\}$



$$dV_{\text{WRN}}(v_m) = \begin{cases} 4 & \text{if } v_m \leq 110 \\ \frac{1}{3} + \frac{1}{30} \cdot v_m & \text{if } 110 < v_m \leq 140 \\ 5 & \text{if } 140 < v_m \end{cases} \quad (1)$$

$$dV_{\text{IL1}}(v_m) = \begin{cases} 5.5 & \text{if } v_m \leq 110 \\ 0.55 + 0.045 \cdot v_m & \text{if } 110 < v_m \leq 210 \\ 10 & \text{if } 210 < v_m \end{cases} \quad (2)$$

$$dV_{\text{IL2}}(v_m) = \begin{cases} 7.5 & \text{if } v_m \leq 110 \\ -0.75 + 0.075 \cdot v_m & \text{if } 110 < v_m \leq 210 \\ 15 & \text{if } 210 < v_m \end{cases} \quad (3)$$

**quiescent pre-state**

$$\mathcal{R} \equiv \bigvee_{i \in \text{IDX}} (\alpha_i \wedge (m, y) = (i, \mathbf{e}_i) \wedge (m', y') = (i, \mathbf{e}_i))$$

$$\bigvee_{(i,j) \in J} (g_{i,j} \wedge (m, y) = (i, \mathbf{e}_i) \wedge (m', y') = (j, \mathbf{e}_j))$$

**transient pre-state**      **quiescent post-state**

$$\text{IDX} = \{1, 2, 3, 4, 5\}$$

$$J = \{(1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 3), (2, 4), (2, 5), \\ (3, 1), (3, 4), (3, 5), (4, 1), (4, 5), (5, 1)\}$$

$$\mathbf{e}_1 = (d = 0, b = 0), \mathbf{e}_2 = (2, 0), \mathbf{e}_3 = (3, 0), \mathbf{e}_4 = \mathbf{e}_5 = (4, 1)$$

$$\begin{aligned}
\alpha_1 &\equiv v \leq v_m \\
\alpha_2 &\equiv v_m < v \wedge v \leq v_m + dV_{\text{WRN}}(v_m) \\
\alpha_3 &\equiv v_m < v \wedge v \leq v_m + dV_{\text{IL1}}(v_m) \\
\alpha_4 &\equiv v_m < v \wedge v \leq v_m + dV_{\text{IL2}}(v_m) \\
\alpha_5 &\equiv (0 < v \wedge a = 0) \vee (v_m < v \wedge a = 1)
\end{aligned}$$

$$g_{1,2} \equiv v_m < v \wedge v \leq v_m + dV_{\text{WRN}}(v_m)$$

$$g_{1,3} \equiv v_m + dV_{\text{WRN}}(v_m) < v \leq v_m + dV_{\text{IL1}}(v_m)$$

$$g_{1,4} \equiv v_m + dV_{\text{IL1}}(v_m) < v \leq v_m + dV_{\text{IL2}}(v_m)$$

$$g_{1,5} \equiv v_m + dV_{\text{IL2}}(v_m) < v$$

$$g_{2,3} \equiv g_{1,3}$$

$$g_{2,4} \equiv g_{3,4} \equiv g_{1,4}$$

$$g_{2,5} \equiv g_{3,5} \equiv g_{4,5} \equiv g_{1,5}$$

$$g_{2,1} \equiv v \leq v_m$$

$$g_{3,1} \equiv g_{4,1} \equiv g_{2,1}$$

$$g_{5,1} \equiv v = 0 \vee (v \leq v_m \wedge a = 1)$$

# Quiescent Reduction

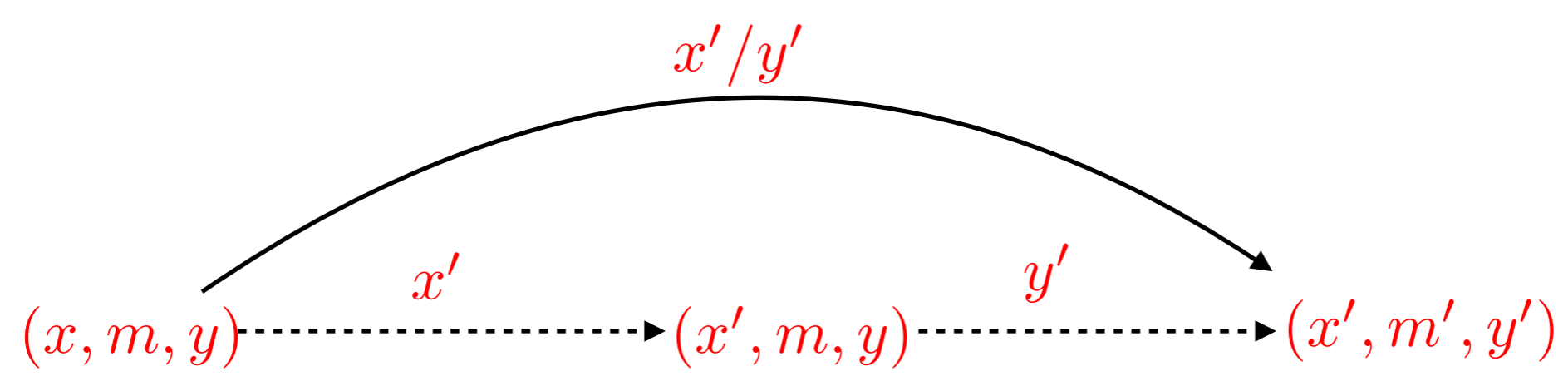
$(S_Q, s_0, R_Q)$  is called quiescent reduction of  $(S, s_0, R)$ :

- $S_Q \subseteq S$  set of quiescent states
- $R_Q \subseteq S_Q \times S_Q, \forall s_1, s_2 \in S_Q, (s_1, s_2) \in R_Q :\Leftrightarrow$ 
  - $(s_1, s_2) \in R$  or
  - $\exists s \in S_T, (s_1, s), (s, s_2) \in R$

$$s_1 \xrightarrow{s_2(x,y)} s_2$$

$R_Q$   $\longrightarrow$

$R$   $\cdots\cdots\cdots\blacktriangleright$



quiescent state

transient state

quiescent state

$R_Q \longrightarrow$

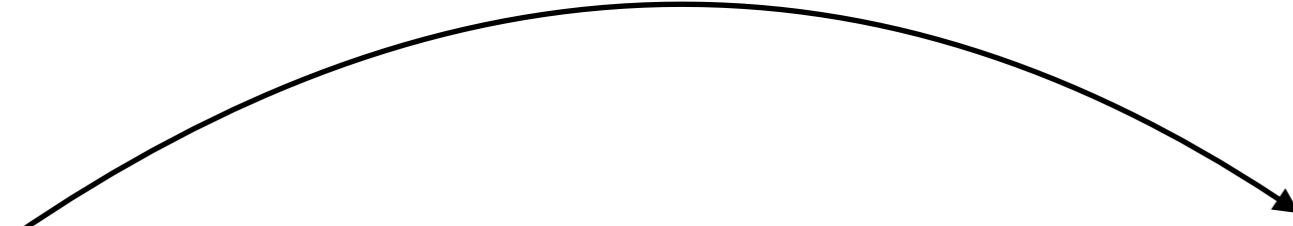
$(x, m, y)$

$x' / y'$

$(x', m', y')$

quiescent state

quiescent state



$s \xrightarrow{(x_1, y_1) \dots (x_k, y_k)} s'$ , if and only if  
 $\exists s, s_1, \dots, s_k = s' \in S_Q$ , such that

1.  $s_i(x, y) = (x_i, y_i), i = 1, \dots, k$

2.  $(s, s_1), (s_i, s_{i+1}) \in R_Q, i = 1, \dots, k - 1$



# Language

$$\mathcal{S} = (S, s_0, R)$$

For any  $s \in S_Q$  quiescent state

- $L(s) := \{\pi \in \Sigma^* \mid \exists s' \in S_Q, s \xrightarrow{\pi} s'\}$  **language of  $s$**
- $s \sim s' :\Leftrightarrow L(s) = L(s')$   **$s$   $s'$  are equivalent**
- $L(\mathcal{S}) := L(s_0)$  **language of  $\mathcal{S}$**

# RIOSTS Properties

$$S_Q = (S_Q, s_0, R_Q)$$

- $S_Q$  is **input-complete** : always holds

$$\forall s \in S_Q \wedge x \in I, \exists y \in O \wedge s' \in S_Q : s \xrightarrow{x/y} s'$$

- $S_Q$  is **deterministic** :

$$\forall s \xrightarrow{x/y_1} s_1, s \xrightarrow{x/y_2} s_2 \in R_Q \Rightarrow y_1 = y_2 \wedge s_1 = s_2$$

# RIOSTS Properties

- $S_Q$  is **observable**, if

$$\forall s \xrightarrow{x/y} s_1, s \xrightarrow{x/y} s_2 \in R_Q \Rightarrow s_1 = s_2$$

- $S_Q$  is **minimal**, if always fails, if  $L$  contains more than two inputs

$$- \forall s \in S_Q, \exists \pi \in L(s_0) : s_0 \xrightarrow{\pi} s$$

$$- \forall s_1 \neq s_2 \in S_Q \Rightarrow L(s_1) \neq L(s_2)$$

# State Equivalence

等價類

$$\mathcal{S} = (S, s_0, I, O, R), s, s' \in S_Q$$

$$s \sim s' :\Leftrightarrow L(s) = L(s') \quad s \ s' \text{ are equivalent}$$

$$[s] = \{s' \in S_Q \mid s \sim s'\} \quad \text{state equivalence class}$$

$$S_Q / \sim = \{[s] \mid s \in S_Q\} \quad \text{equivalence class partition}$$

finite ? or infinite ?

When is  $S_Q/\sim$  finite?

sufficient condition :  $M$  and  $O$  are finite.

$$\begin{aligned} & s_1 = (x_1, m_1, y_1), s_2 = (x_2, m_2, y_2) \in S_Q \wedge (m_1, y_1) = (m_2, y_2) \\ & \Rightarrow \forall x \in I, s = (x, m_1, y_1) \in S : (s_1, s), (s_2, s) \in R \\ & \Rightarrow \forall x \in I, s' = (x, m', y') \in S_Q : ((s_1, s') \in R_Q \Leftrightarrow (s_2, s') \in R_Q) \\ & \Rightarrow L(s_1) = L(s_2) \end{aligned}$$

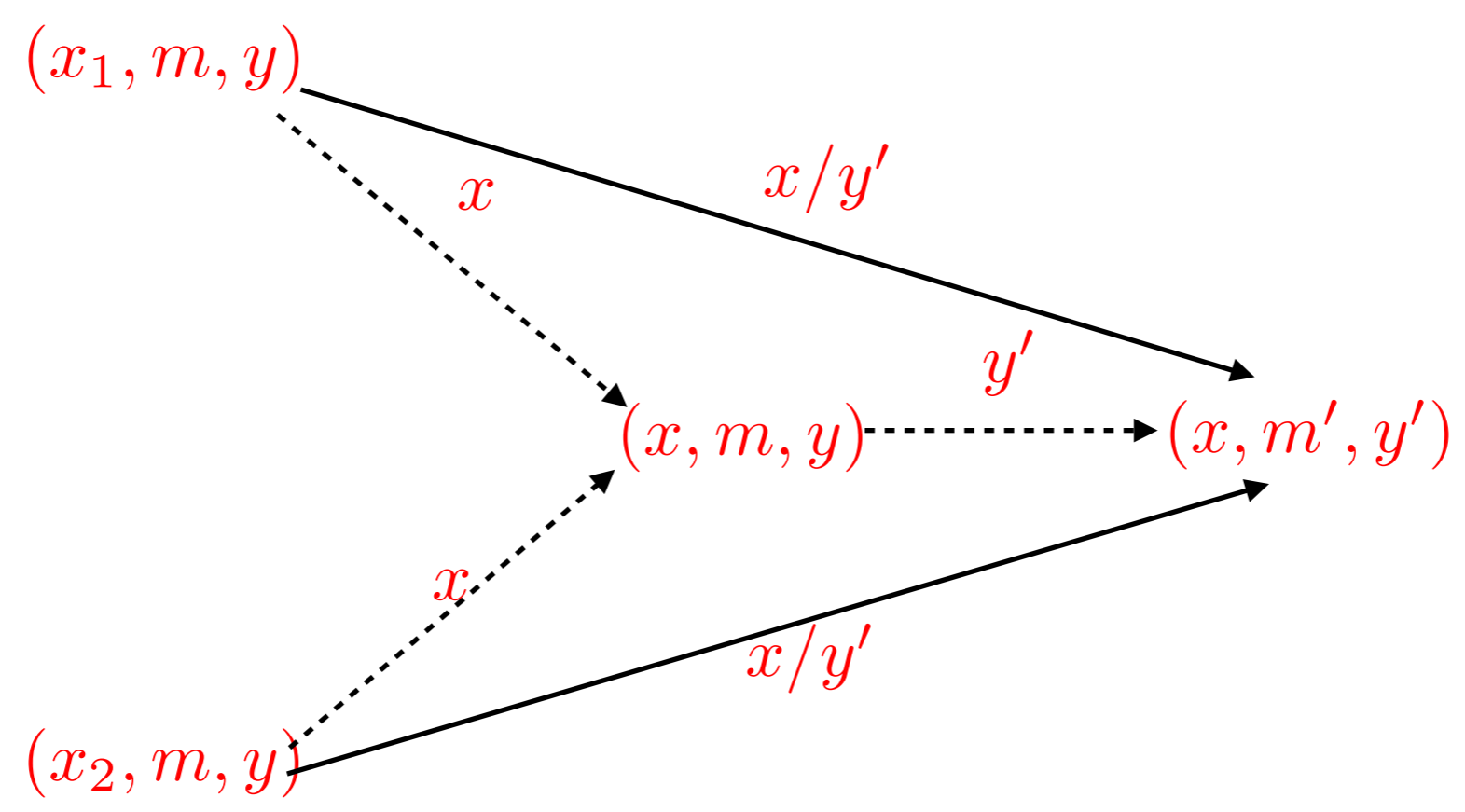
$$M \times O =: \{(m_1, y_1), \dots, (m_n, y_n)\}, n \leq |M| \cdot |O|$$

$$A_i := \{s \in S_Q \mid s(m, y) = (m_i, y_i)\}, i = 1, \dots, n$$

$\{A_i \mid i = 1, \dots, n\}$  is a refinement of  $S_Q/\sim$

$R_Q \longrightarrow$

$R \cdots \longrightarrow$



$$\mathcal{R} \equiv \bigvee_{i \in \text{IDX}} (\alpha_i \wedge (m, y) = (m_i, y_i) \wedge (m', y') = (m_i, y_i))$$

$$\vee \bigvee_{(i,j) \in J} (g_{i,j} \wedge (m, y) = (m_i, y_i) \wedge (m', y') = (m_j, y_j))$$

$$A_i = \{s \in S \mid s(\alpha_i) \wedge s(m) = m_i \wedge s(y) = y_i\}$$

Signature  $Sig_1$  is a set of

**deterministic** RIOSTS with

- finite outputs
- finite internal state values



Let  $S_Q/\sim = \{[s_1], \dots, [s_n]\}$  and  $O = \{y_1, \dots, y_\ell\}$

The **transition index function**

$$\delta : I \rightarrow \{1, \dots, n\}^n; \quad x \mapsto (\delta_1(x), \dots, \delta_n(x))$$

$$\delta_i(x) = j \Leftrightarrow \forall s \in [s_i], \exists s' \in [s_j], s \xrightarrow{x/y} s'$$

The **output index function**

$$\omega : I \rightarrow \{1, \dots, \ell\}^n; \quad x \mapsto (\omega_1(x), \dots, \omega_n(x))$$

$$\omega_i(x) = k \Leftrightarrow \forall s \in [s_i], \exists s' \in S_Q, s \xrightarrow{x/y_k} s'$$

The **transition index function**

$$\delta : I \rightarrow \{1, \dots, n\}^n; \quad x \mapsto (\delta_1(x), \dots, \delta_n(x))$$

$$\delta_i(x) = j \Leftrightarrow \forall s \in [s_i], \exists s' \in [s_j], s \xrightarrow{x/y} s'$$

and the **output index function**

$$\omega : I \rightarrow \{1, \dots, \ell\}^n; \quad x \mapsto (\omega_1(x), \dots, \omega_n(x))$$

$$\omega_i(x) = k \Leftrightarrow \forall s \in [s_i], x/y_k \in L(s)$$

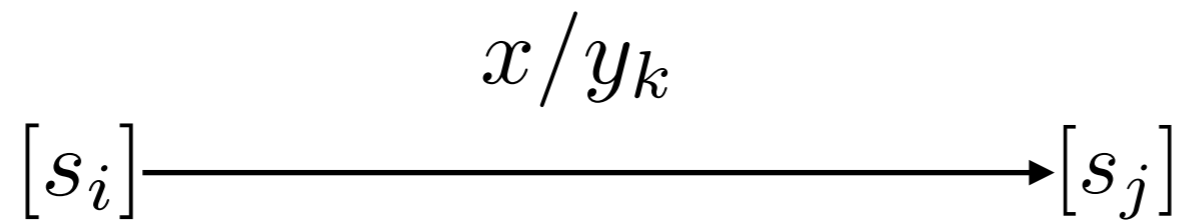
are well-defined:

$$s \sim u \Leftrightarrow L(s) = L(u) \Leftrightarrow \forall x \in I:$$

$$(s \xrightarrow{x/y} s', u \xrightarrow{x/y'} u' \Rightarrow s' \sim u' \wedge y = y')$$

Let  $S_Q/\sim = \{[s_1], \dots, [s_n]\}$  and  $O = \{y_1, \dots, y_\ell\}$

$$\delta_i(x) = j \wedge \omega_i(x) = k$$



# Input Equivalence

$$\mathcal{S} = (S, s_0, I, O, R), x, x' \in I$$

$$x \sim x' :\Leftrightarrow \delta(x) = \delta(x') \wedge \omega(x) = \omega(x') \quad x \ x' \text{ are equivalent}$$

$$[x] = \{x' \in I \mid x \sim x'\} \quad \text{input equivalence class}$$

$$I/\sim = \{[x] \mid x \in I\} \quad \text{input equivalence class partition}$$

finite ? or infinite ?

$$[x] = \{x' \in I \mid x \sim x'\} = \{x' \in I \mid \delta(x') = \delta(x) \wedge \omega(x') = \omega(x)\}$$

$$\delta(x) \in \{1, \dots, n\}^n, \omega(x) \in \{1, \dots, \ell\}^n$$

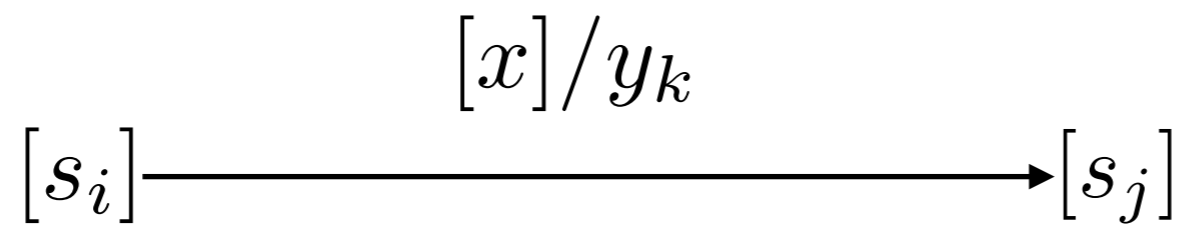
$$\{(\delta(x), \omega(x)) \mid x \in I\} \subseteq \{1, \dots, n\}^n \times \{1, \dots, \ell\}^n$$

$$|\{(\delta(x), \omega(x)) \mid x \in I\}| \leq |\{1, \dots, n\}^n \times \{1, \dots, \ell\}^n| = n^n \cdot \ell^n$$

$$|I/\sim| = |\{(\delta(x), \omega(x)) \mid x \in I\}| \leq n^n \cdot \ell^n \text{ is finite !}$$

Let  $S_Q/\sim = \{[s_1], \dots, [s_n]\}$  and  $O = \{y_1, \dots, y_\ell\}$

$$\delta_i(x) = j \wedge \omega_i(x) = k$$



# Model Map

$Sig_1$ : deterministic RIOSTS with  $|O|, |M|$  are finite.

$Sig_2$ : input completed, minimal and deterministic FSMs.

$\mathcal{I}$ : any refinement of  $I/\sim$ .

$$\begin{aligned} T : Sig_1 &\rightarrow Sig_2 \\ (S, s_0, R) &\mapsto (S_Q/\sim, [s_0], \mathcal{I}, O, h) \end{aligned}$$

$$[s_1] \xrightarrow{[x]/y} [s_2] \in h$$

$$\Leftrightarrow \exists s \in [s_1], s' \in [s_2], x_1 \in [x]_{\mathcal{I}} : s \xrightarrow{x_1/y} s' \in R_Q$$

# Satisfaction Condition (1)

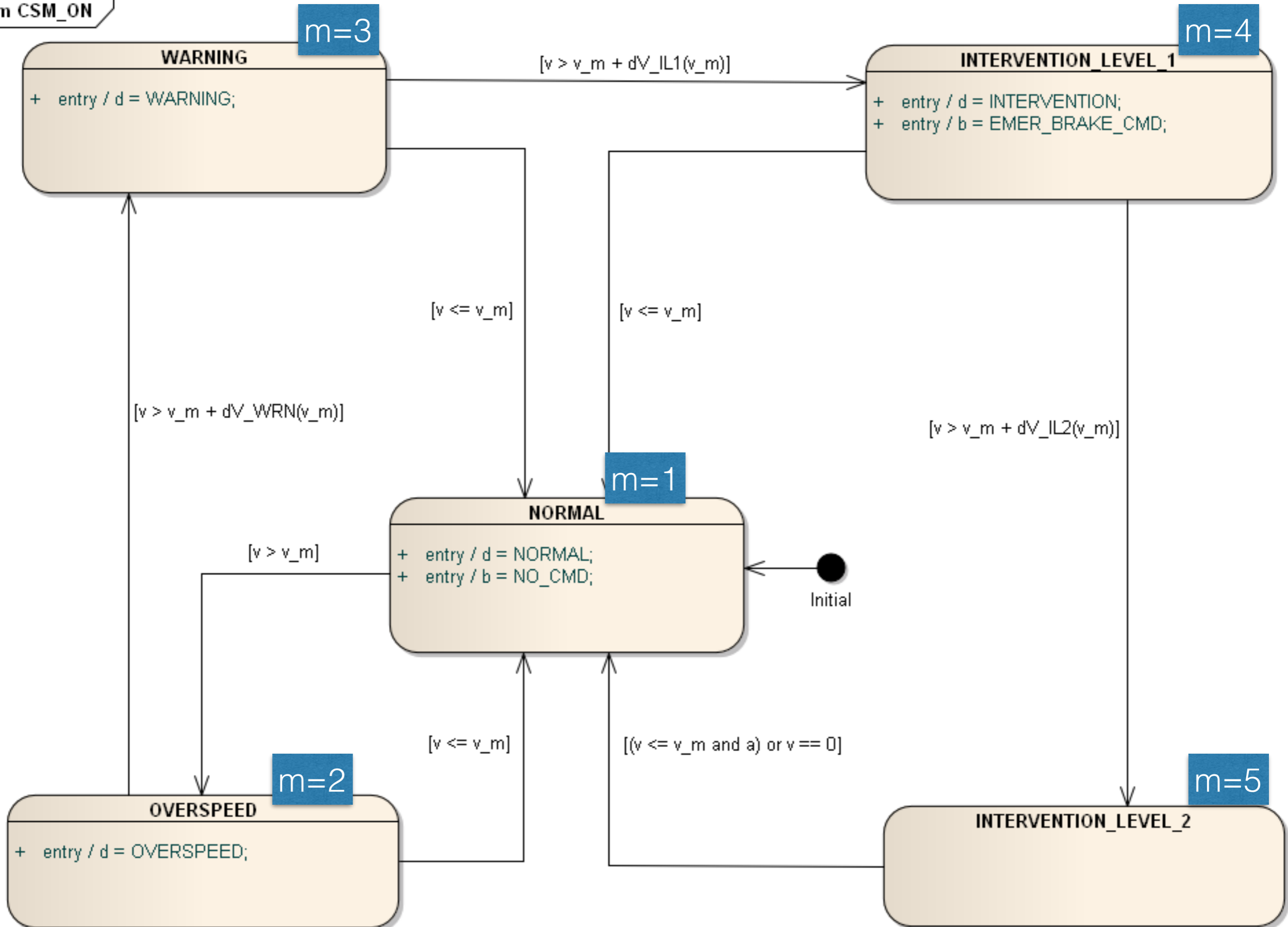
$$(T, T^*) : \mathcal{F}(Sig_1) \times \mathbf{tc}(Sig_2) \not\rightarrow \mathcal{F}(Sig_2) \times \mathbf{tc}(Sig_1)$$
$$\mathcal{F}(S, \leq_1, \mathcal{D}_1) \xrightarrow{T} \mathcal{F}(T(S), \leq_2, \mathcal{D}_2)$$

$$\mathcal{D}_1 = \{S' \in Sig_1 \mid \mathcal{I} \text{ is a refinement of } I/\sim'\}$$

$$\begin{array}{ccc} S & \longrightarrow & T(S) \\ \leq_1 \downarrow & & \downarrow \leq_2 \\ S' & \longrightarrow & T(S') \end{array}$$



stm CSM\_ON



- $x = (v, v_m, a) \in I, I = [0, 350] \times [0, 350] \times \{0, 1\}$
- $m \in \{1, 2, 3, 4, 5\}$
- $y = (d, b) \in O, O = \{0, 2, 3, 4\} \times \{0, 1\}$

$$y_1 = (d = 0, b = 0), y_2 = (2, 0),$$

$$y_3 = (3, 0), y_4 = y_5 = (4, 1)$$

$$A_i = \{s \in \mathcal{S} \mid s(\alpha_i) \wedge s(m) = m_i \wedge s(y) = y_i\}$$

$$\Phi_1 \equiv g_{1,1} \wedge g_{2,1} \wedge g_{3,1} \wedge g_{4,1} \wedge g_{5,5}$$

$$\equiv 0 < v \leq v_m \wedge a = 0$$

$$\Phi_2 \equiv g_{1,1} \wedge g_{2,1} \wedge g_{3,1} \wedge g_{4,1} \wedge g_{5,1}$$

$$\equiv v = 0 \vee (v \leq v_m \wedge a = 1)$$

$$\Phi_3 \equiv g_{1,2} \wedge g_{2,2} \wedge g_{3,3} \wedge g_{4,4} \wedge g_{5,5}$$

$$\equiv v_m < v \leq v_m + dV_{\text{WRN}}(v_m)$$

$$\Phi_4 \equiv g_{1,3} \wedge g_{2,3} \wedge g_{3,3} \wedge g_{4,4} \wedge g_{5,5}$$

$$\equiv v_m + dV_{\text{WRN}}(v_m) < v \leq v_m + dV_{\text{IL1}}(v_m)$$

$$\Phi_5 \equiv g_{1,4} \wedge g_{2,4} \wedge g_{3,4} \wedge g_{4,4} \wedge g_{5,5}$$

$$\equiv v_m + dV_{\text{IL1}}(v_m) < v \leq v_m + dV_{\text{IL2}}(v_m)$$

$$\Phi_6 \equiv g_{1,5} \wedge g_{2,5} \wedge g_{3,5} \wedge g_{4,5} \wedge g_{5,5}$$

$$\equiv v_m + dV_{\text{IL2}}(v_m) < v$$

Define

$$X_i = \{(v, v_m, a) \in I \mid (v, v_m, a) \models \Phi_i\} \quad i = 1, \dots, 6$$

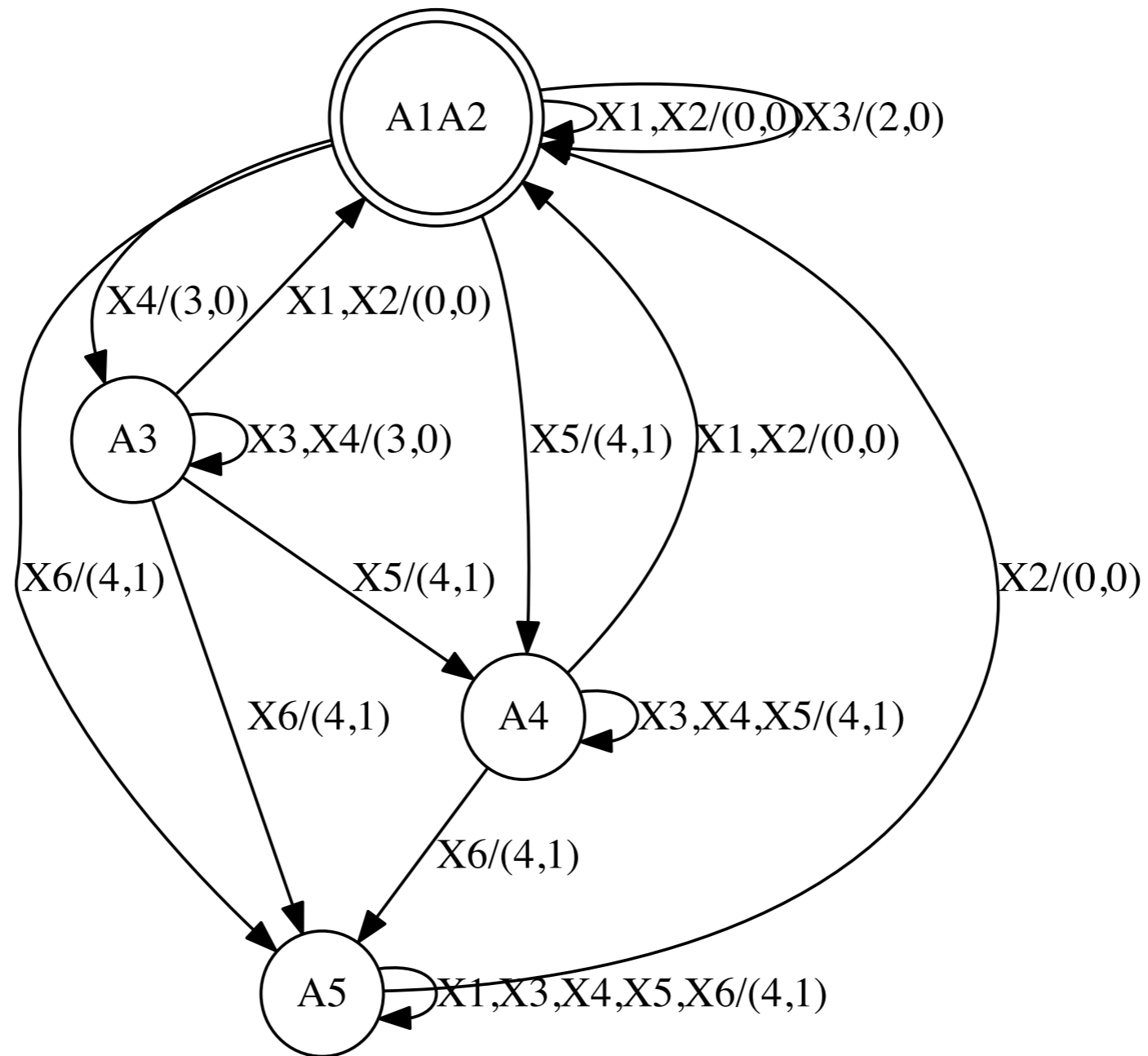
$\mathcal{I} = \{X_1, X_2, \dots, X_6\}$ . Then  $\mathcal{I}$  is an IECP for the CSM

Table 1: Input Representatives of  $X_i$ .

$\mathbf{c}_i$	$v$	$v_m$	$a$	$X_i$
$\mathbf{c}_1$	60	90	0	$X_1$
$\mathbf{c}_2$	60	90	1	$X_2$
$\mathbf{c}_3$	152	150	0	$X_3$
$\mathbf{c}_4$	125	120	1	$X_4$
$\mathbf{c}_5$	66	60	0	$X_5$
$\mathbf{c}_6$	260	230	0	$X_6$

Table 1: DFSM Transition Table.

Source	Input	Target	d	b
$A_1$	$X_1 \cup X_2$	$A_1$	0	0
$A_1$	$X_3$	$A_2$	2	0
$A_1$	$X_4$	$A_3$	3	0
$A_1$	$X_5$	$A_4$	4	1
$A_1$	$X_6$	$A_5$	4	1
$A_2$	$X_1 \cup X_2$	$A_1$	0	0
$A_2$	$X_3$	$A_2$	2	0
$A_2$	$X_4$	$A_3$	3	0
$A_2$	$X_5$	$A_4$	4	1
$A_2$	$X_6$	$A_5$	4	1
$A_3$	$X_1 \cup X_2$	$A_1$	0	0
$A_3$	$X_3 \cup X_4$	$A_3$	3	0
$A_3$	$X_5$	$A_4$	4	1
$A_3$	$X_6$	$A_5$	4	1
$A_4$	$X_1 \cup X_2$	$A_1$	0	0
$A_4$	$X_3 \cup X_4 \cup X_5$	$A_4$	4	1
$A_4$	$X_6$	$A_5$	4	1
$A_5$	$X_2$	$A_1$	0	0
$A_5$	$\bigcup_{i \in \{1,3,4,5,6\}} X_i$	$A_5$	4	1



$$L(S) = \{\pi \in \Sigma^* \mid [\pi] \in L(T(S))\}$$

# Test Case Map

$$T^* : \text{tc of } Sig_2 \rightarrow \text{tc of } Sig_1$$

- A test case for deterministic FSM is a finite input/output sequence.
- A test case for nondeterministic FSM is considered as a FSM,  
—”preset” or ”adaptive” test case.



- A test case for deterministic RIOSTS is a finite input/output sequence.
- A test case for nondeterministic RIOSTS is considered as an RIOSTS with input alphabet  $O$  and output alphabet  $I$ .

# Test Case Map

$$T^* : \text{tc of } Sig_2 \rightarrow \text{tc of } Sig_1$$

$$\begin{array}{ccc} (S, s_0, I, O, R) & \xrightarrow{T} & (S_Q/\sim, [s_0], \mathcal{I}, O, h) \\ X_1/y_1 \dots X_K/y_k & \xrightarrow{T^*} & x_1/y_1 \dots x_k/y_k \end{array}$$

$\mathcal{I}$  is a refinement of  $I/\sim$  and  $[x_i]_{\mathcal{I}} = X_i, i = 1, \dots, k$

# Satisfaction Condition (2)

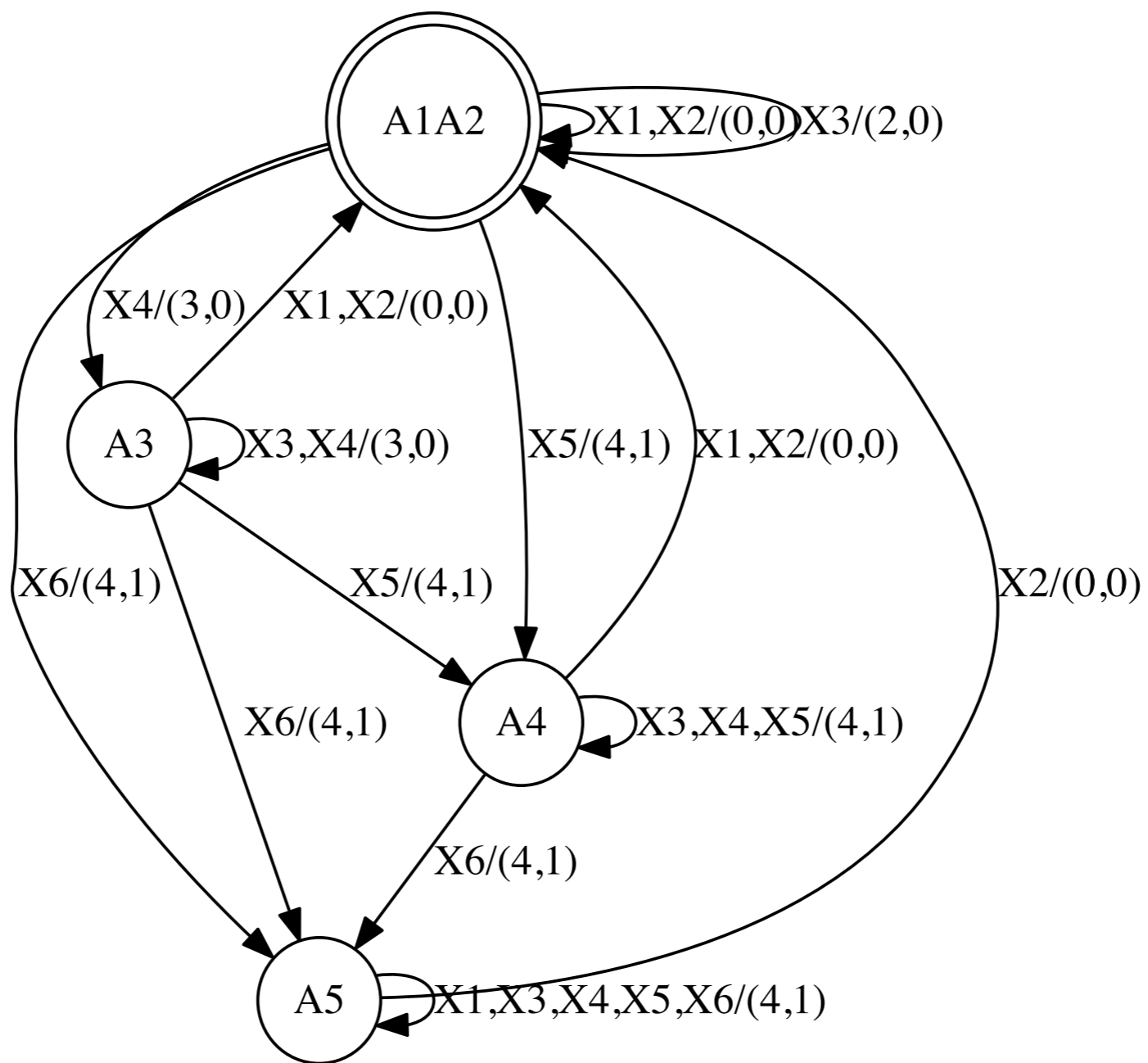
$$(T, T^*) : \mathcal{F}(\text{Sig}_1) \times \mathbf{tc}(\text{Sig}_2) \not\rightarrow \mathcal{F}(\text{Sig}_2) \times \mathbf{tc}(\text{Sig}_1)$$
$$\mathcal{F}(S, \leq_1, \mathcal{D}_1) \xrightarrow{T} \mathcal{F}(T(S), \leq_2, \mathcal{D}_2)$$

- $\forall S' \in \mathcal{D}_1, U \in \mathbf{tc}(\mathcal{I}, \mathcal{O}) : T^*(U) \in \mathbf{tc}(\mathcal{I}, \mathcal{O})$
- $T(S') \underline{\text{pass}}_2 U \Leftrightarrow S' \underline{\text{pass}}_1 T^*(U)$

$$\begin{array}{ccc} S' & \longrightarrow & T(S') \\ \underline{\text{pass}}_1 \downarrow & & \downarrow \underline{\text{pass}}_2 \\ T^*(U) & \longleftarrow & U \end{array}$$

# Application of Theorem 2

1. Calculate **input equivalence classes**
2. **Map speed monitor model to FSM** with
  1. input equivalence classes as input alphabet
  2. original discrete outputs as output alphabet
3. Use W-Method or similar method to **create complete FSM test suite**
4. **Translate FSM test suite** to concrete test suite for speed monitor



# Symbolic test cases resulting from W-Method

1.  $x_4 \cdot x_1 \cdot x_3 / (3, 0) \cdot (0, 0) \cdot (2, 0)$
2.  $x_4 \cdot x_1 \cdot x_1 / (3, 0) \cdot (0, 0) \cdot (0, 0)$
3.  $x_4 \cdot x_2 \cdot x_3 / (3, 0) \cdot (0, 0) \cdot (2, 0)$
4.  $x_4 \cdot x_2 \cdot x_1 / (3, 0) \cdot (0, 0) \cdot (0, 0)$
5.  $x_4 \cdot x_3 \cdot x_3 / (3, 0) \cdot (3, 0) \cdot (3, 0)$
6.  $x_4 \cdot x_3 \cdot x_1 / (3, 0) \cdot (3, 0) \cdot (0, 0)$
7.  $x_4 \cdot x_4 \cdot x_3 / (3, 0) \cdot (3, 0) \cdot (3, 0)$
8.  $x_4 \cdot x_4 \cdot x_1 / (3, 0) \cdot (3, 0) \cdot (0, 0)$
9.  $x_4 \cdot x_5 \cdot x_3 / (3, 0) \cdot (4, 1) \cdot (4, 1)$
10.  $x_4 \cdot x_5 \cdot x_1 / (3, 0) \cdot (4, 1) \cdot (0, 0)$
11.  $x_4 \cdot x_6 \cdot x_3 / (3, 0) \cdot (4, 1) \cdot (4, 1)$
12.  $x_4 \cdot x_6 \cdot x_1 / (3, 0) \cdot (4, 1) \cdot (4, 1)$
13.  $x_5 \cdot x_1 \cdot x_3 / (4, 1) \cdot (0, 0) \cdot (2, 0)$
14.  $x_5 \cdot x_1 \cdot x_1 / (4, 1) \cdot (0, 0) \cdot (0, 0)$
15.  $x_5 \cdot x_2 \cdot x_3 / (4, 1) \cdot (0, 0) \cdot (2, 0)$
16.  $x_5 \cdot x_2 \cdot x_1 / (4, 1) \cdot (0, 0) \cdot (0, 0)$
17.  $x_5 \cdot x_3 \cdot x_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
18.  $x_5 \cdot x_3 \cdot x_1 / (4, 1) \cdot (4, 1) \cdot (0, 0)$
19.  $x_5 \cdot x_4 \cdot x_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
20.  $x_5 \cdot x_4 \cdot x_1 / (4, 1) \cdot (4, 1) \cdot (0, 0)$
21.  $x_5 \cdot x_5 \cdot x_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
22.  $x_5 \cdot x_5 \cdot x_1 / (4, 1) \cdot (4, 1) \cdot (0, 0)$
23.  $x_5 \cdot x_6 \cdot x_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
24.  $x_5 \cdot x_6 \cdot x_1 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
25.  $x_6 \cdot x_1 \cdot x_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
26.  $x_6 \cdot x_1 \cdot x_1 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
27.  $x_6 \cdot x_2 \cdot x_3 / (4, 1) \cdot (0, 0) \cdot (2, 0)$
28.  $x_6 \cdot x_2 \cdot x_1 / (4, 1) \cdot (0, 0) \cdot (0, 0)$
29.  $x_6 \cdot x_3 \cdot x_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
30.  $x_6 \cdot x_3 \cdot x_1 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
31.  $x_6 \cdot x_4 \cdot x_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
32.  $x_6 \cdot x_4 \cdot x_1 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
33.  $x_6 \cdot x_5 \cdot x_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
34.  $x_6 \cdot x_5 \cdot x_1 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
35.  $x_6 \cdot x_6 \cdot x_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
36.  $x_6 \cdot x_6 \cdot x_1 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
37.  $x_1 \cdot x_3 / (0, 0) \cdot (2, 0)$
38.  $x_1 \cdot x_1 / (0, 0) \cdot (0, 0)$
39.  $x_2 \cdot x_3 / (0, 0) \cdot (2, 0)$
40.  $x_2 \cdot x_1 / (0, 0) \cdot (0, 0)$
41.  $x_3 \cdot x_3 / (2, 0) \cdot (2, 0)$
42.  $x_3 \cdot x_1 / (2, 0) \cdot (0, 0)$

# Symbolic test cases resulting from W-Method

1.	$X_4.X_1.X_3 / (2, 0) \cdot (0, 0) \cdot (2, 0)$	21.	$X_5.X_5.X_2 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
2.	$X_4.X_1.X_2 / (4, 1) \cdot (4, 1) \cdot (0, 0)$		
3.	$X_4.X_2.X_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$		
4.	$X_4.X_2.X_2 / (4, 1) \cdot (4, 1) \cdot (4, 1)$		
5.	$X_4.X_3.X_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$		
6.	$X_4.X_3.X_2 / (4, 1) \cdot (4, 1) \cdot (4, 1)$		
7.	$X_4.X_4.X_3 / (4, 1) \cdot (0, 0) \cdot (2, 0)$		
8.	$X_4.X_4.X_2 / (4, 1) \cdot (0, 0) \cdot (0, 0)$		
9.	$X_4.X_5.X_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$		
10.	$X_4.X_5.X_2 / (4, 1) \cdot (4, 1) \cdot (4, 1)$		
11.	$X_4.X_6.X_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$		
12.	$X_4.X_6.X_2 / (4, 1) \cdot (4, 1) \cdot (4, 1)$		
13.	$X_5.X_1.X_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$		
14.	$X_5.X_1.X_2 / (4, 1) \cdot (4, 1) \cdot (4, 1)$		
15.	$X_5.X_2.X_3 / (4, 1) \cdot (0, 0) \cdot (2, 0)$	35.	$X_6.X_6.X_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
16.	$X_5.X_2.X_1 / (4, 1) \cdot (0, 0) \cdot (0, 0)$	36.	$X_6.X_6.X_1 / (4, 1) \cdot (4, 1) \cdot (4, 1)$
17.	$X_5.X_3.X_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$	37.	$X_1.X_3 / (0, 0) \cdot (2, 0)$
18.	$X_5.X_3.X_1 / (4, 1) \cdot (4, 1) \cdot (0, 0)$	38.	$X_1.X_1 / (0, 0) \cdot (0, 0)$
19.	$X_5.X_4.X_3 / (4, 1) \cdot (4, 1) \cdot (4, 1)$	39.	$X_2.X_3 / (0, 0) \cdot (2, 0)$
20.	$X_5.X_4.X_1 / (4, 1) \cdot (4, 1) \cdot (0, 0)$	40.	$X_2.X_1 / (0, 0) \cdot (0, 0)$
		41.	$X_3.X_3 / (2, 0) \cdot (2, 0)$
		42.	$X_3.X_1 / (2, 0) \cdot (0, 0)$

**Symbolic** means that concrete test data still has to be selected from each  $X_i$  when it is referenced in a test case

This can be done automatically using a mathematical constraint solver (**SMT-solver**)

# Combination With Random and Boundary Value Testing

- Instead of always using the same representative of each input class representative, **select a random value of this class**, whenever it is used in the test case – combine this technique with **boundary value tests**
- Completeness is still guaranteed for SUTs inside the fault domain
- **For SUTs outside the fault domain, the test strength is significantly increased**

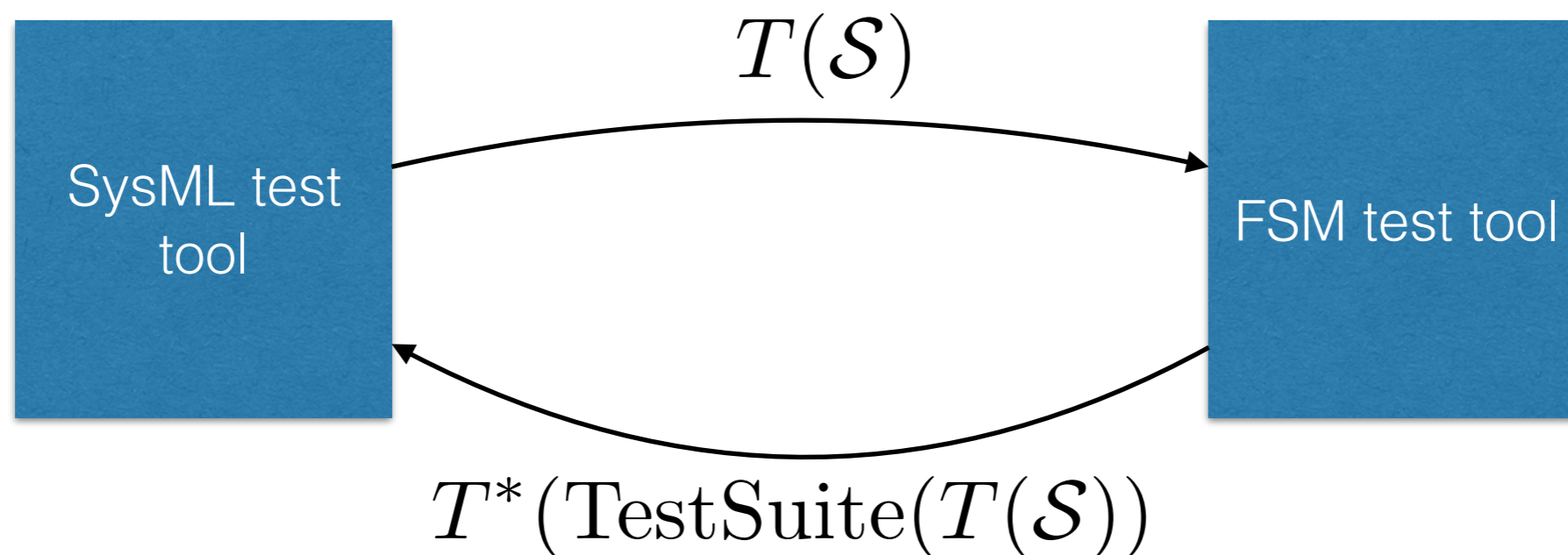


# Summary of the Benefits

- A new complete testing strategy for systems with infinite input domains and finite internal states and finite outputs
- Effectively implementable in model-based testing tools – fully automated
- Significantly **higher test strength** compared to heuristic test strategies
- Significant **reduction of test effort** in application domains where the testing is very costly: railway interlocking systems

# Summary of the Benefits

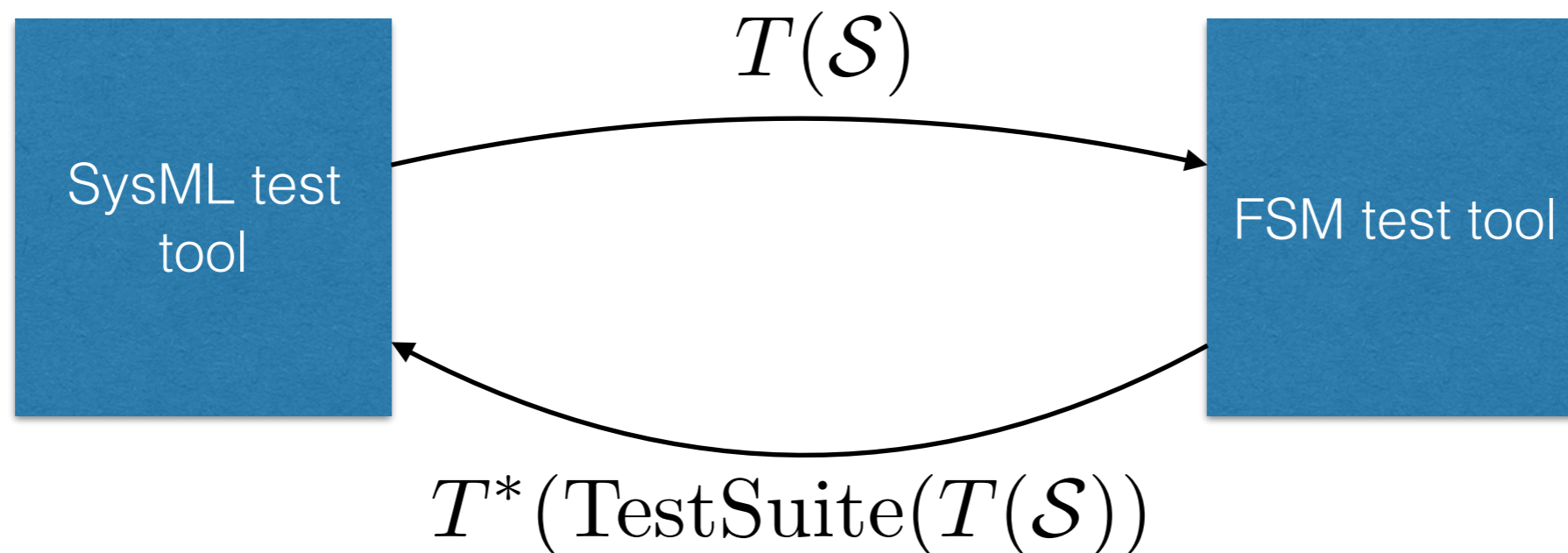
- When building a new tool for model-based testing of SysML state machines (with infinite input domains), the test case generation can be performed by an existing tool implementing these algorithms for FSMs



# Summary of the Benefits

- When building a new tool for model-based testing of SysML state machines (with conceptually infinite input domains), the test performed by an existing algorithms for FSMs

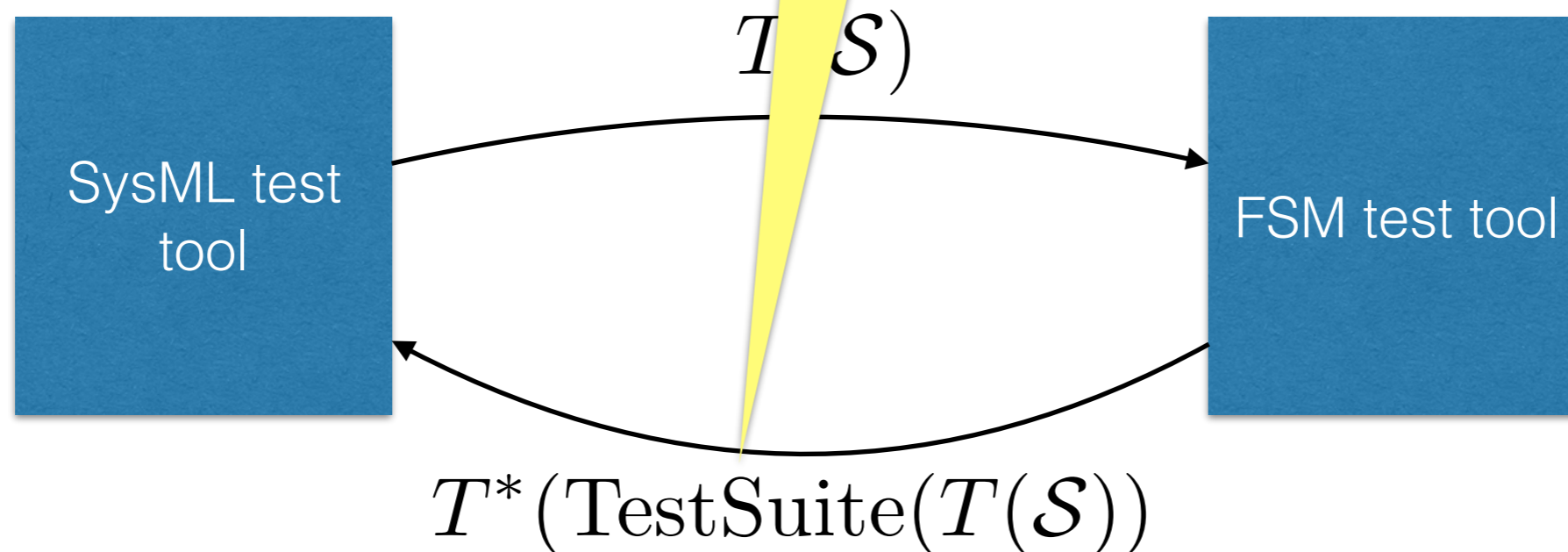
SysML test tool performs FSM abstraction and sends it to FSM test tool



# Summary of the Benefits

- When building a new tool for model-based testing of SysML state machines (with conceptually infinite input domains), the test performed by an existing algorithms for FSMs

FSM tool generates complete test suite and sends the translated result to SysML test tool



# Further Reading

1. Publications of Jan Peleska, Wen-ling Huang, and their co-authors. [http://www.informatik.uni-bremen.de/agbs/jp/jp\\_papers\\_e.html](http://www.informatik.uni-bremen.de/agbs/jp/jp_papers_e.html)
2. ERTMS/ETCS SystemRequirements Specification, Chapter 3, Principles, volume Subset-026-3, Issue 3.4.0 (2015), available under <http://www.era.europa.eu/Document-Register/Pages/Set-2-System-Requirements-Specification.aspx>
3. Nancy Leveson. SafeWare: System Safety and Computers. Addison Wesley 1995.
4. Neil Storey. Safety-critical Computer Systems. Addison-Welly, 1996.