

Testing Safety-critical Discrete- State Systems – Mathematical Foundations and Concrete Algorithms

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Background

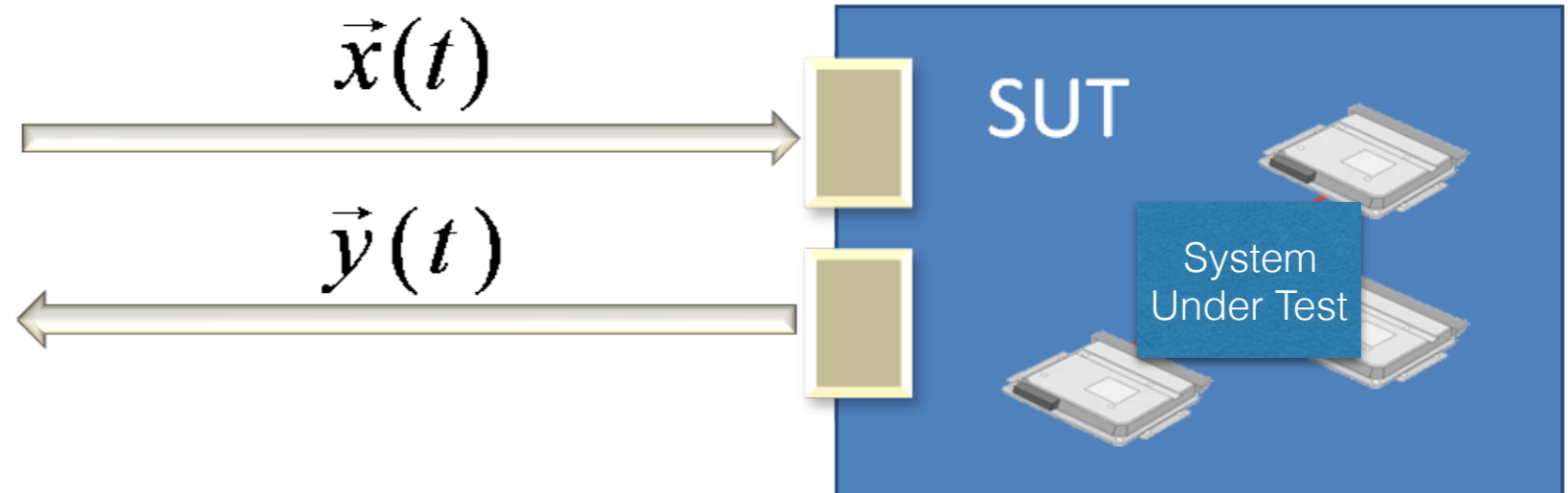
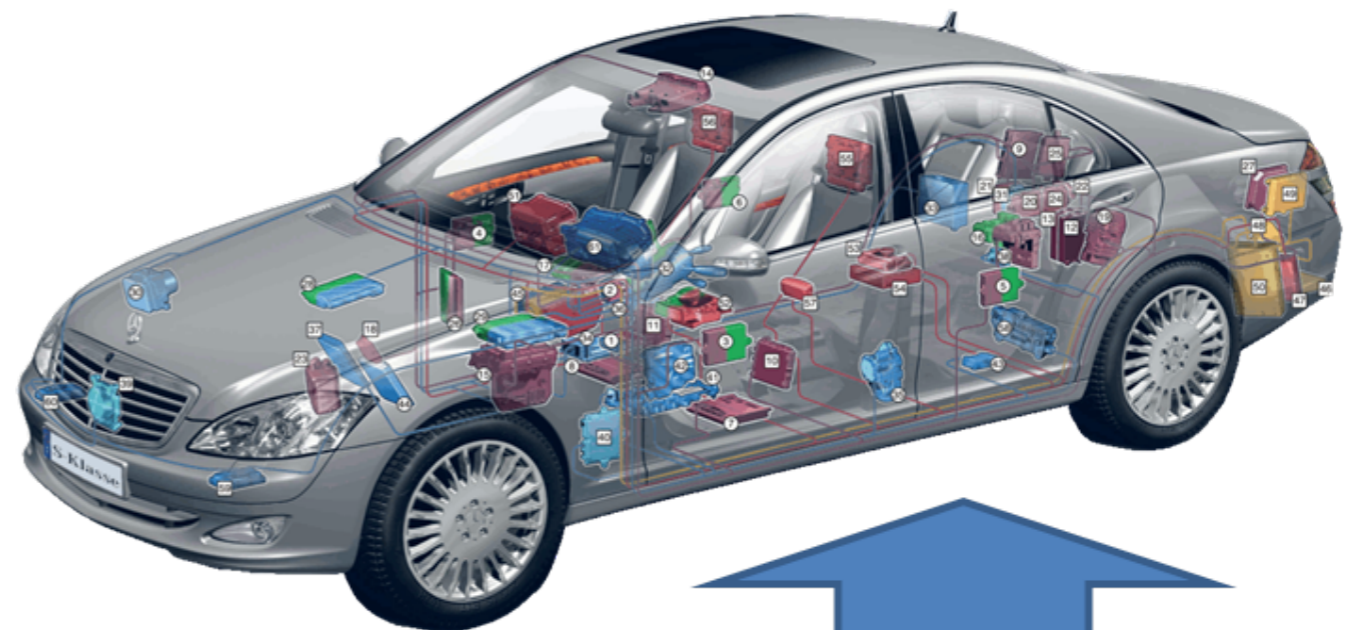
- **My research group at the University of Bremen**
 - Specialised on verification of safety-critical control systems
 - Focus on test automation
 - Collaboration with Prof. Dr. habil. Wen-ling Huang in this field

Background

- **Verified Systems International GmbH**
 - Founded 1998 as a spinoff company from the University of Bremen
 - Specialised on verification and validation of safety-critical systems – aerospace, railways, automotive
 - Tool development, hardware-in-the-loop test bench development, and service provision
 - Main customers Airbus, Siemens
 - 25 employees

Hardware-in-the-Loop Test Benches

For testing integrated HW/SW systems



An Observation . . .

- Many maths and computer science students in their first semesters seem to think that complex theory and difficult algorithms are only needed to pass exams . . .
- This is not true!
- Many of the most important innovations and products are based on highly complex mathematical foundations and on sophisticated software algorithms

Safety-critical systems

- Examples

Safety-critical Systems

A . . . **safety-critical system** is a system whose failure or malfunction may result in one (or more) of the following outcomes:

- death or serious injury to people
- loss or severe damage to equipment/property
- environmental harm

Safety-critical Systems – Examples

Airbag controller



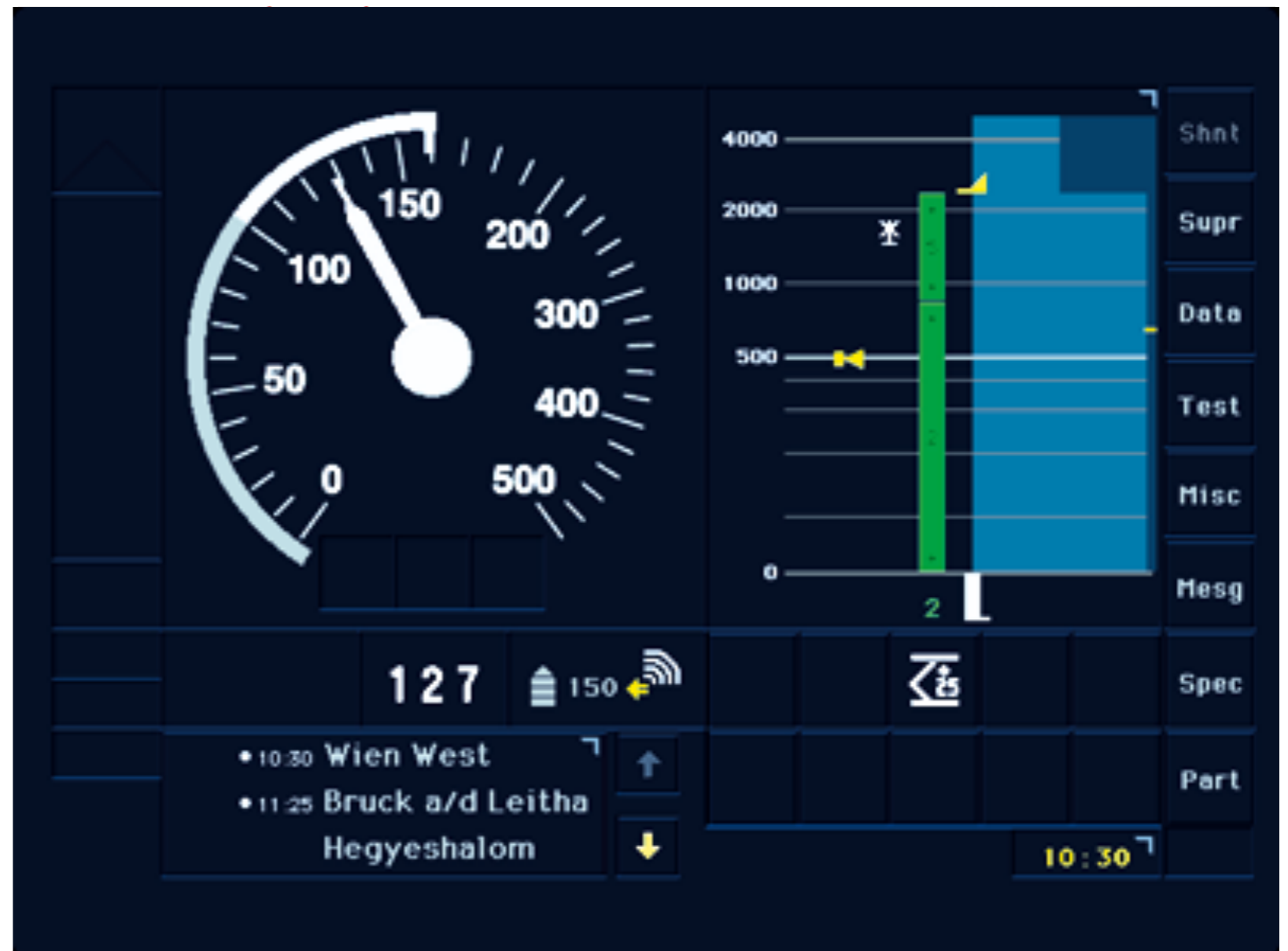
Safety-critical Systems – Examples

Aircraft thrust
reversal



Safety-critical Systems – Examples

Train speed
supervision



Verification Requirements of International Standards

Verification Requirements of International Standards

- Safety-critical systems development and verification is controlled by laws
- These laws state that development and verification must follow the rules specified in applicable standards, such as
 - Avionic domain: **RTCA DO-178C**
 - Railway domain: **CENELEC EN50128:2011**
 - Automotive domain: **ISO 26262**

Verification Requirements of International Standards

- Safety-critical systems development and verification is controlled by laws
- These **Avionic System**. A control verification must computer in an aircraft able standards, such
- Avionic domain: **RTCA DO-178C**
- Railway domain: **CENELEC EN50128:2011**
- Automotive domain: **ISO 26262**

Verification Requirements of International Standards

- These standards differ in many details, but they contain some basic requirements
 - Development must be based on **requirements**
 - Every piece of software code and every hardware component must be **traced** back to at least one requirement

- For the most critical applications, requirements should be expressed by **formal models** with mathematical interpretation
- **Syntax and static model semantics**: is the model well-formed?
- **Behavioural model semantics**: how does the model state, including inputs and outputs, change over time?
- **Requirements (models) must be verified**

- **Code must be verified**
 - Does it implement the related requirements correctly?
 - Verification is preferably performed by **testing** the software integrated in the controller's hardware
- Verification results need to be checked with respect to completeness and correctness
- Tools automating development or verification steps need to be **qualified**

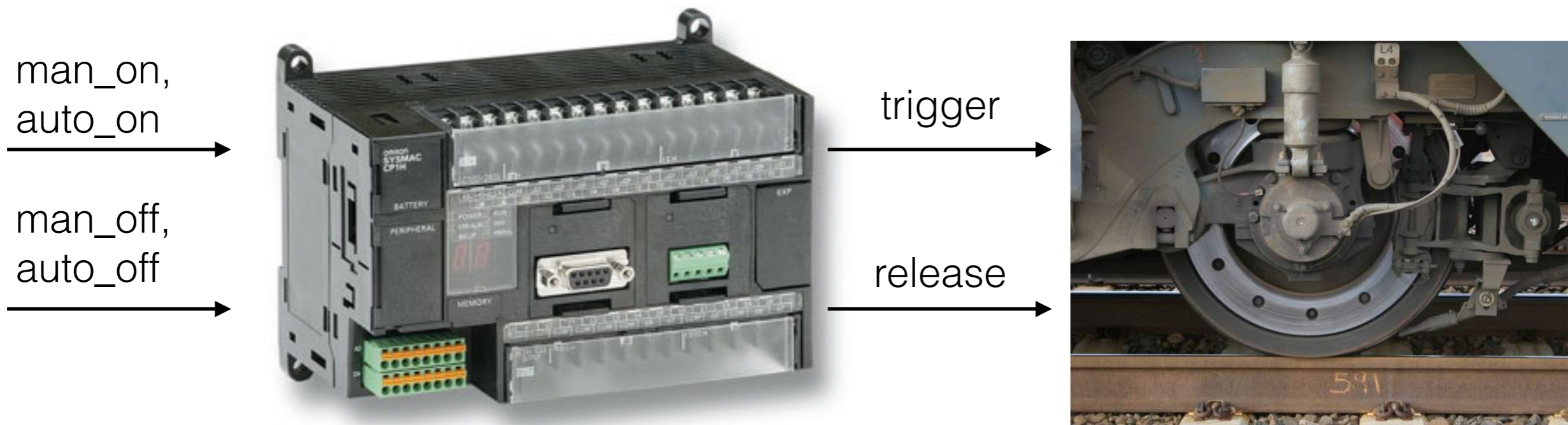
Motivation

- **Safety-critical systems can be developed by starting with a **model** of the required system behaviour**
- **The developed system can then be tested against the model**
 - **Input data to be exercised on the **system under test (SUT)** can be derived from the model**
 - **The behaviour of the SUT can be compared to the **expected behaviour** specified in the model**
 - **<https://zh.wikipedia.org/wiki/生命攸關系統#.E9.81.8B.E8.BC.B8>**

Motivation

- **Testing theories**
 - Define methods to generate **test cases** from models
 - **Test case.** Sequence of input data + expected outputs to produced by the SUT when receiving these inputs
 - Specify which correctness properties are fulfilled if all test cases are passed by the SUT
- For safety-critical systems, the **test strength** (= the capability to uncover certain types of errors) of a testing theory must be proven

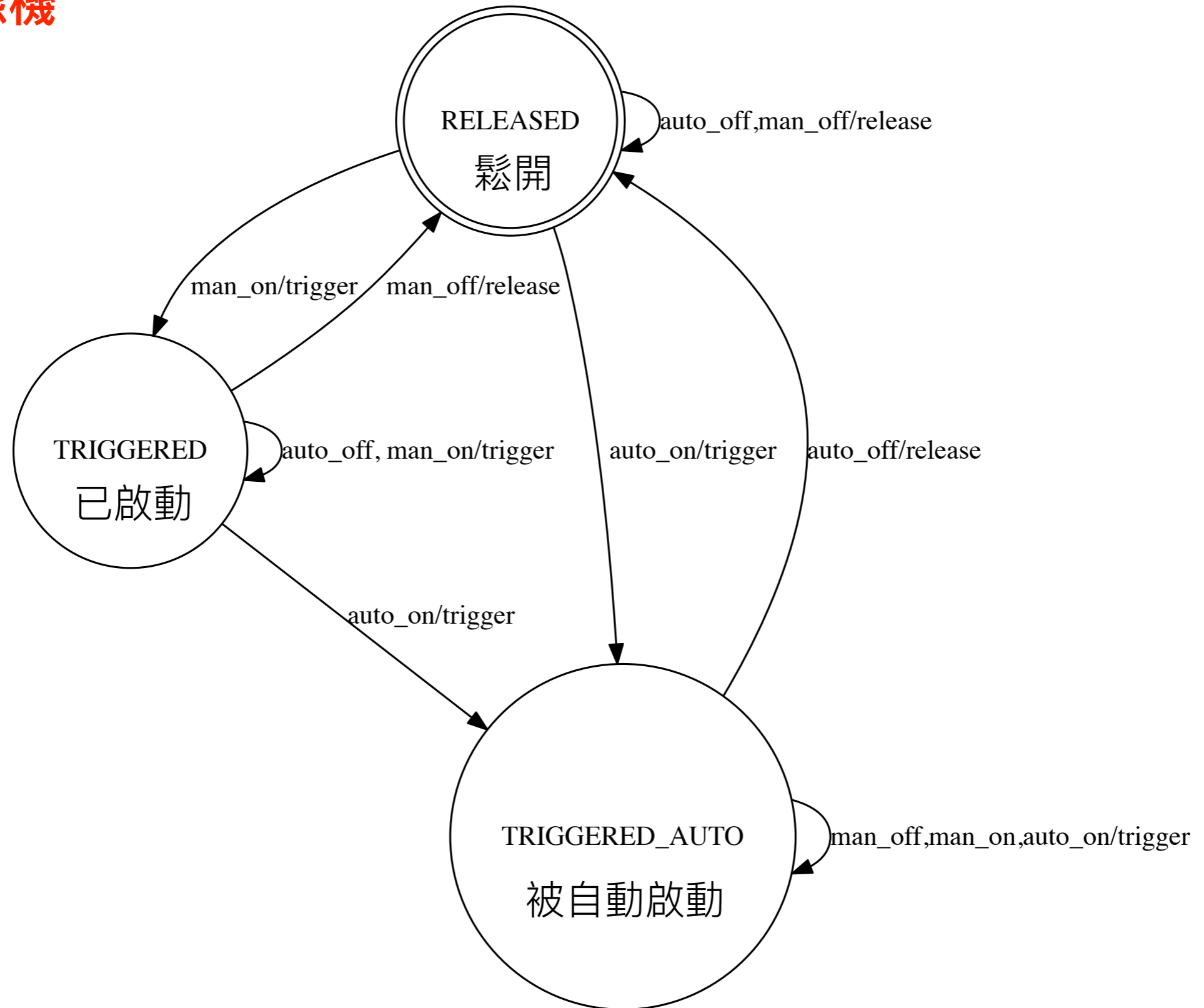
- **Example: Model of a brake controller in a train**
 - **Controller receives commands to activate brakes**
 - **From train engine driver: *man_on, man_off***
 - **From a speed supervision computer: *auto_on, auto_off***
 - **Controller activates brakes by outputs *trigger* and *release***



- **Requirements for the brake controller**
 - 1. When brakes are not activated, they can be triggered manually (*man_on*) or by the speed supervision computer (*auto_on*)**
 - 2. When brakes have been activated manually (*man_on*), they can only be released manually (*man_off*)**
 - 3. When brakes have been activated via speed supervision computer (*auto_on*), they can be only released via command *auto_off* from the computer**
 - 4. When the supervision computer sends *auto_on* while already braking, the brakes can only be released by *auto_off***

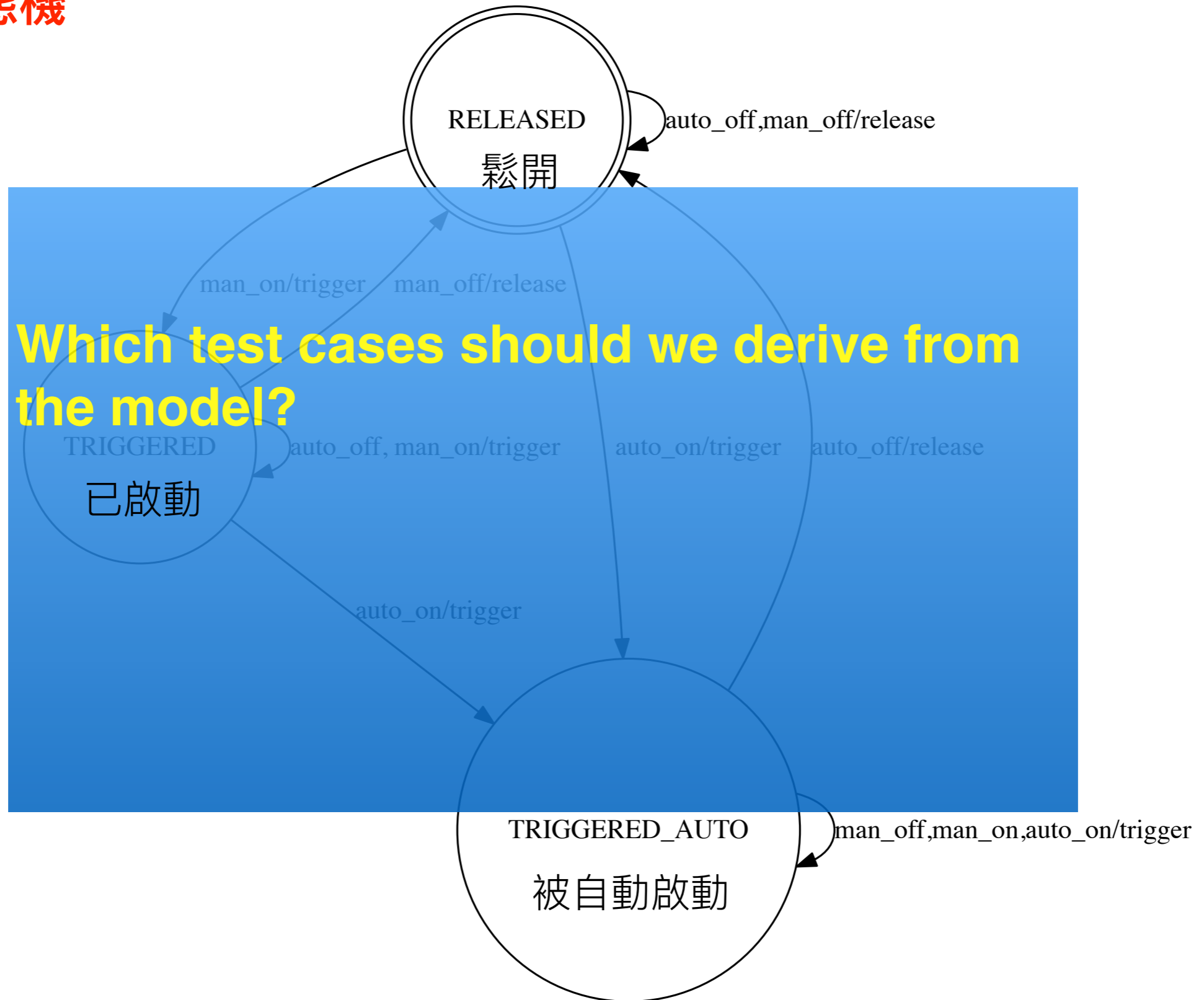
Finite State Machine modelling the behaviour of the brake controller

有限狀態機



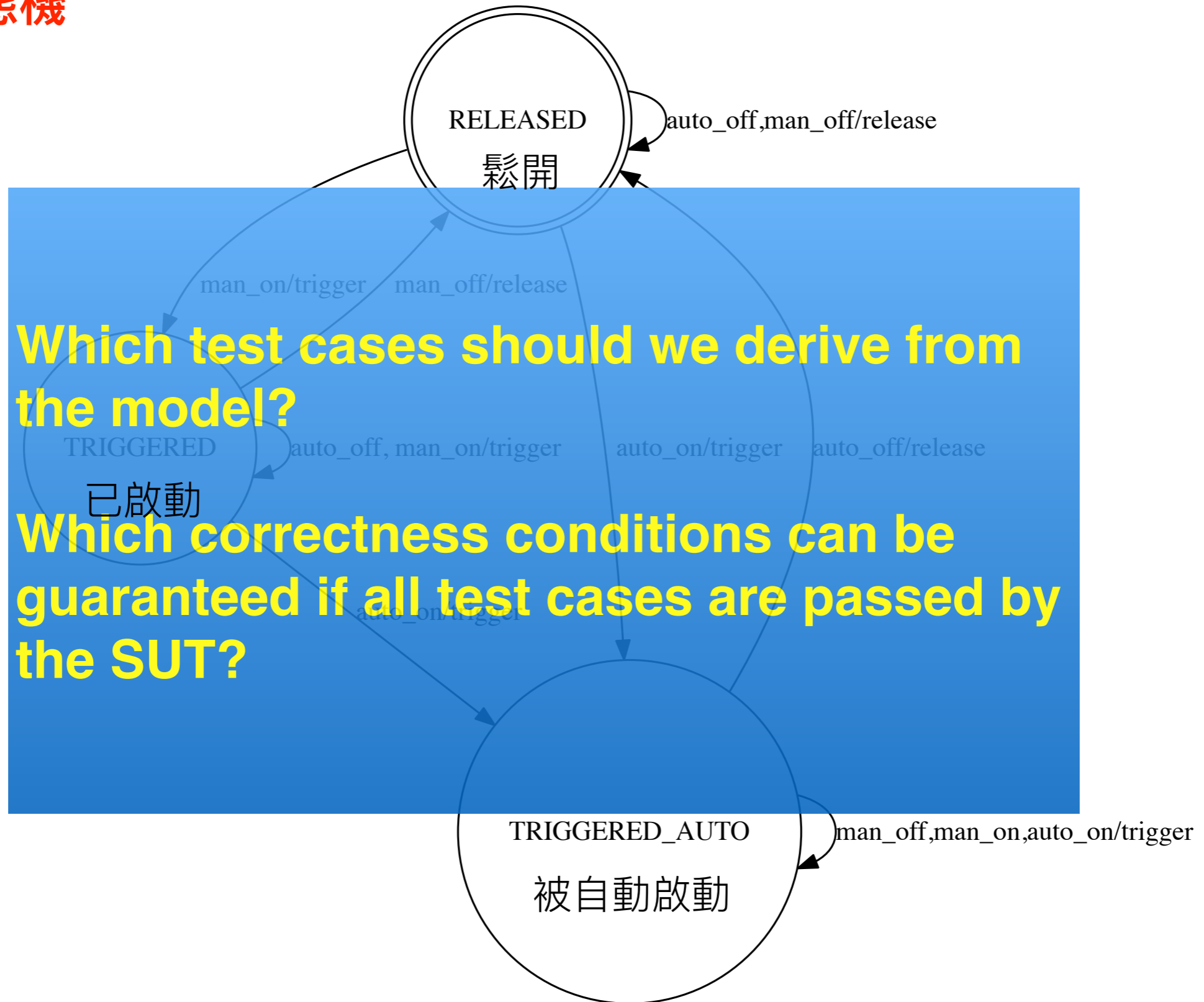
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Overview

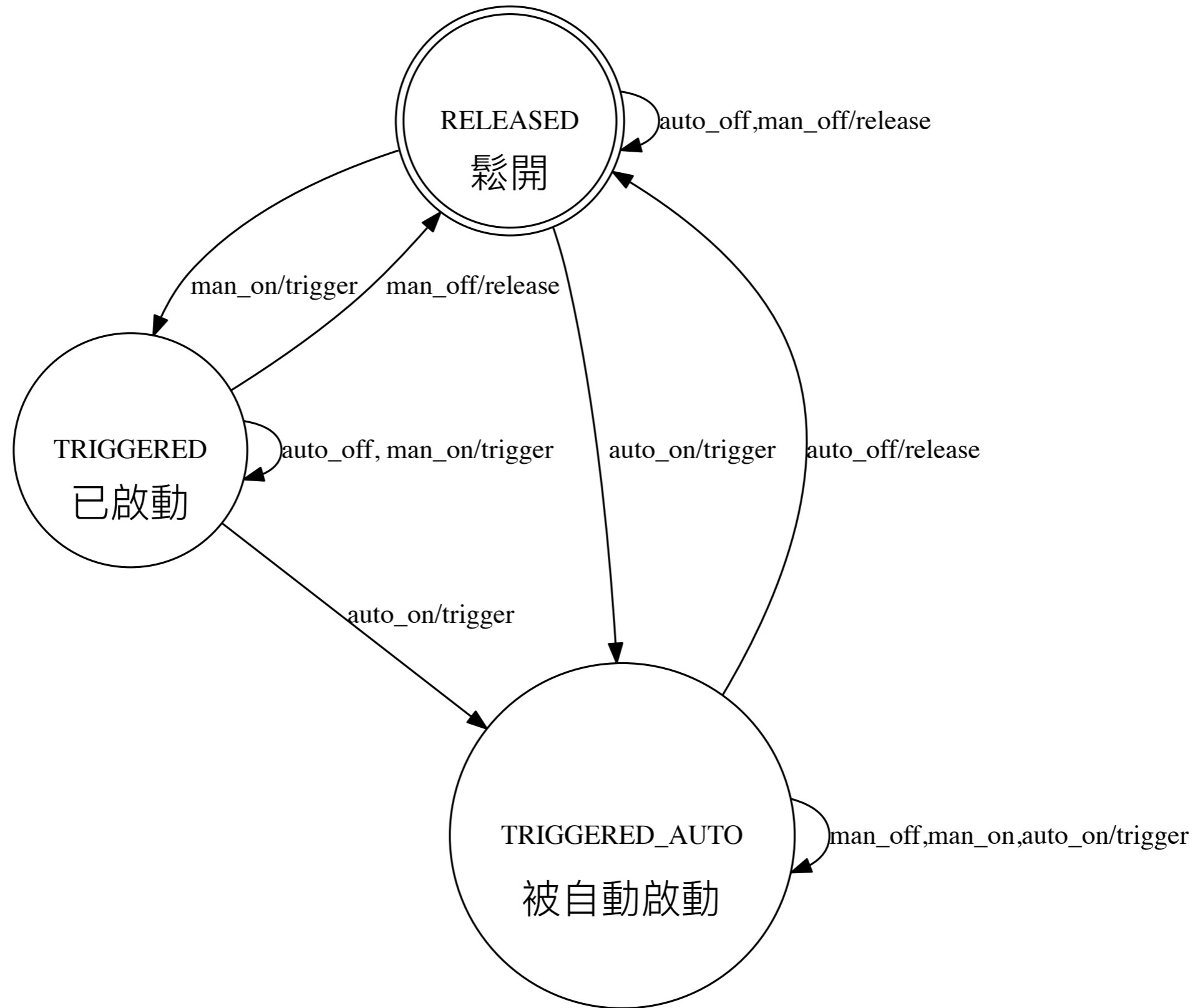
- **Mathematical definition of Finite State Machines**
- **Languages and Conformance Relations**
- **FSM properties: observability and minimality**
- **Fault Models**
- **Testing Theories**
 - **The W-Method for deterministic FSMs**
 - **The Wp-Method for nondeterministic FSMs**

Finite State Machines

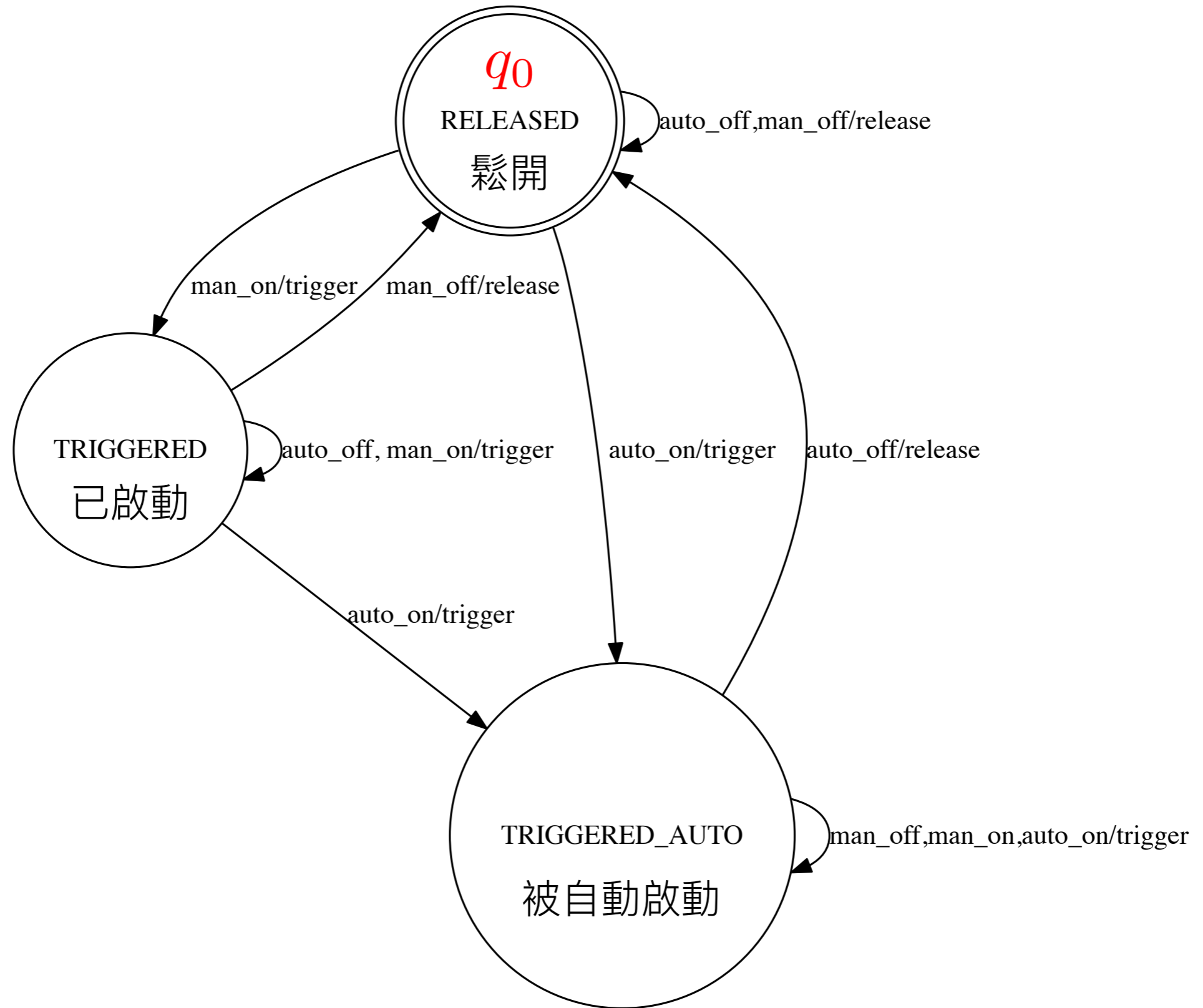
$$M = (Q, q_0, I, O, h)$$

- $Q \neq \emptyset$: finite set of states 狀態集
- $q_0 \in Q$: initial state 初始狀態
- $I \neq \emptyset$: finite set of input alphabet 輸入字母表
- $O \neq \emptyset$: finite set of output alphabet 輸出字母表
- $h \subseteq Q \times I \times O \times Q$: transition relation 狀態遷移關係

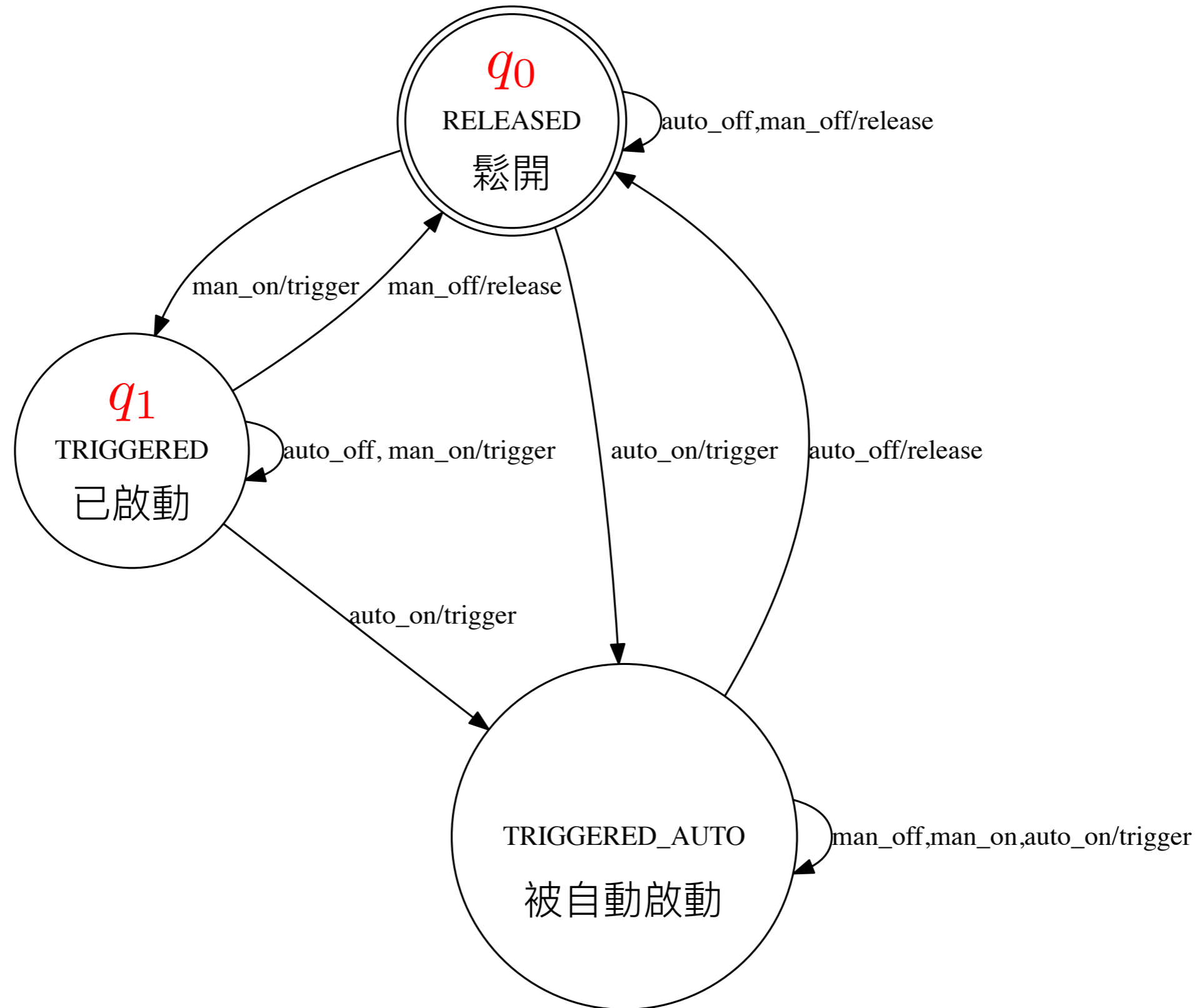
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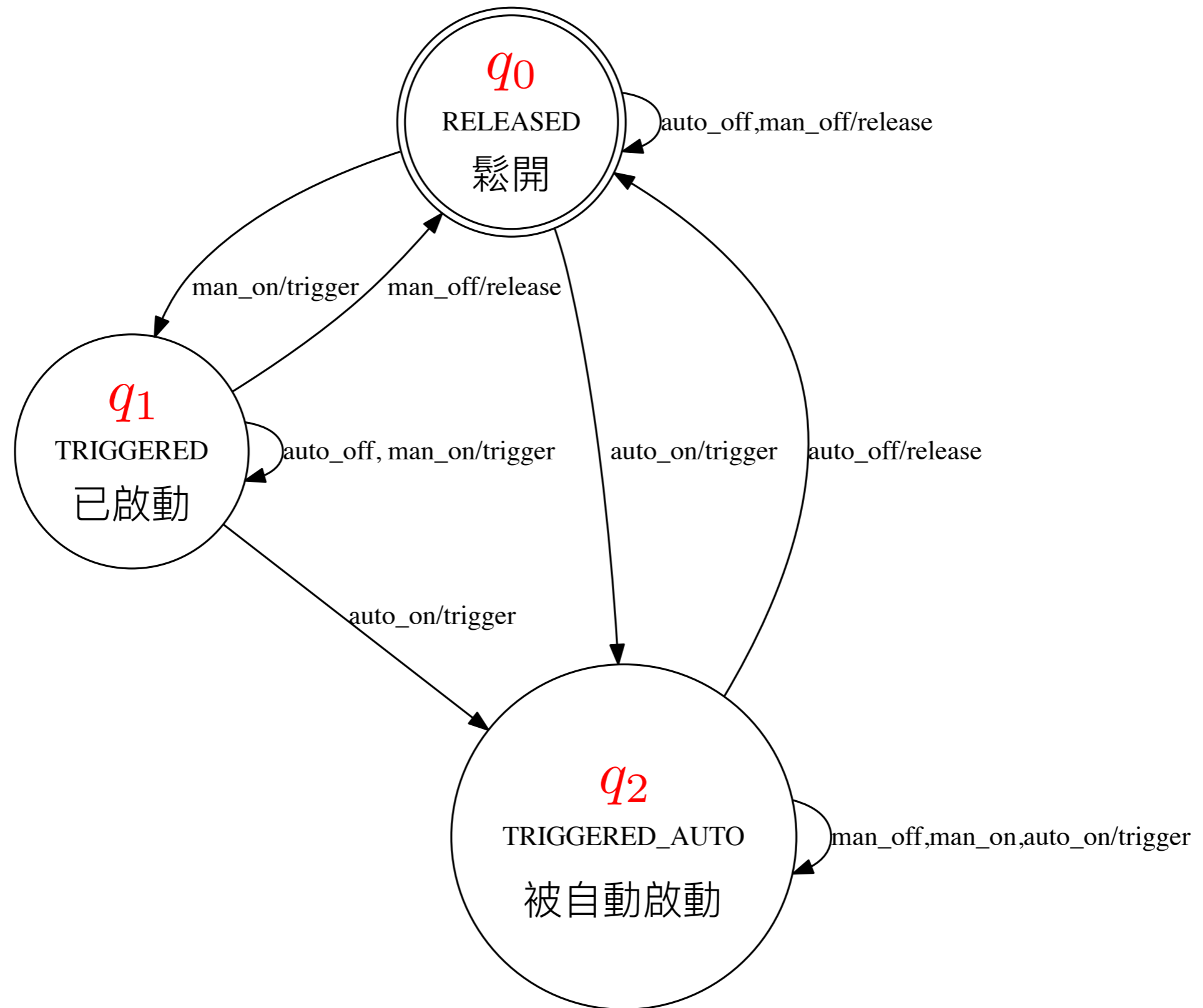
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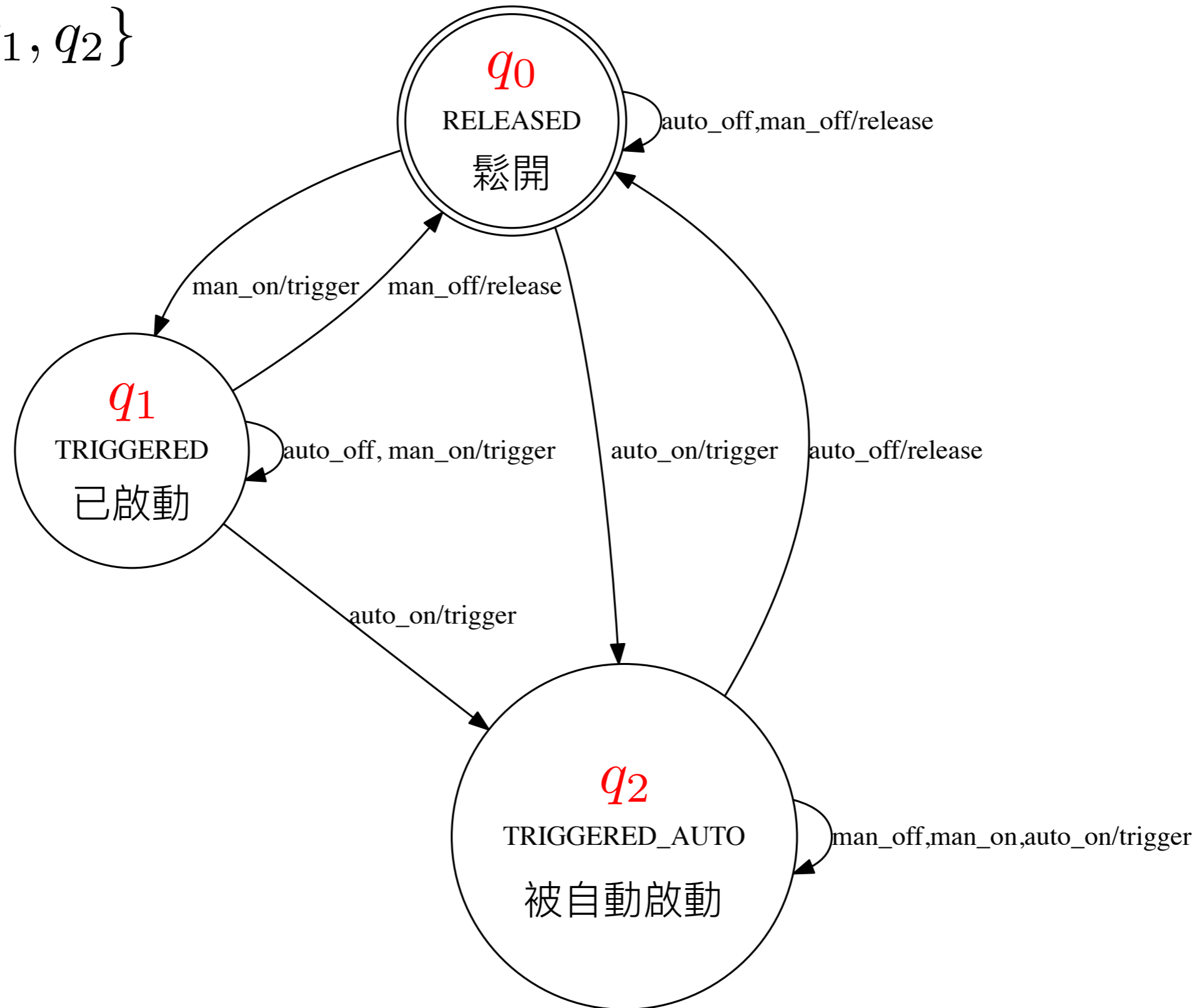


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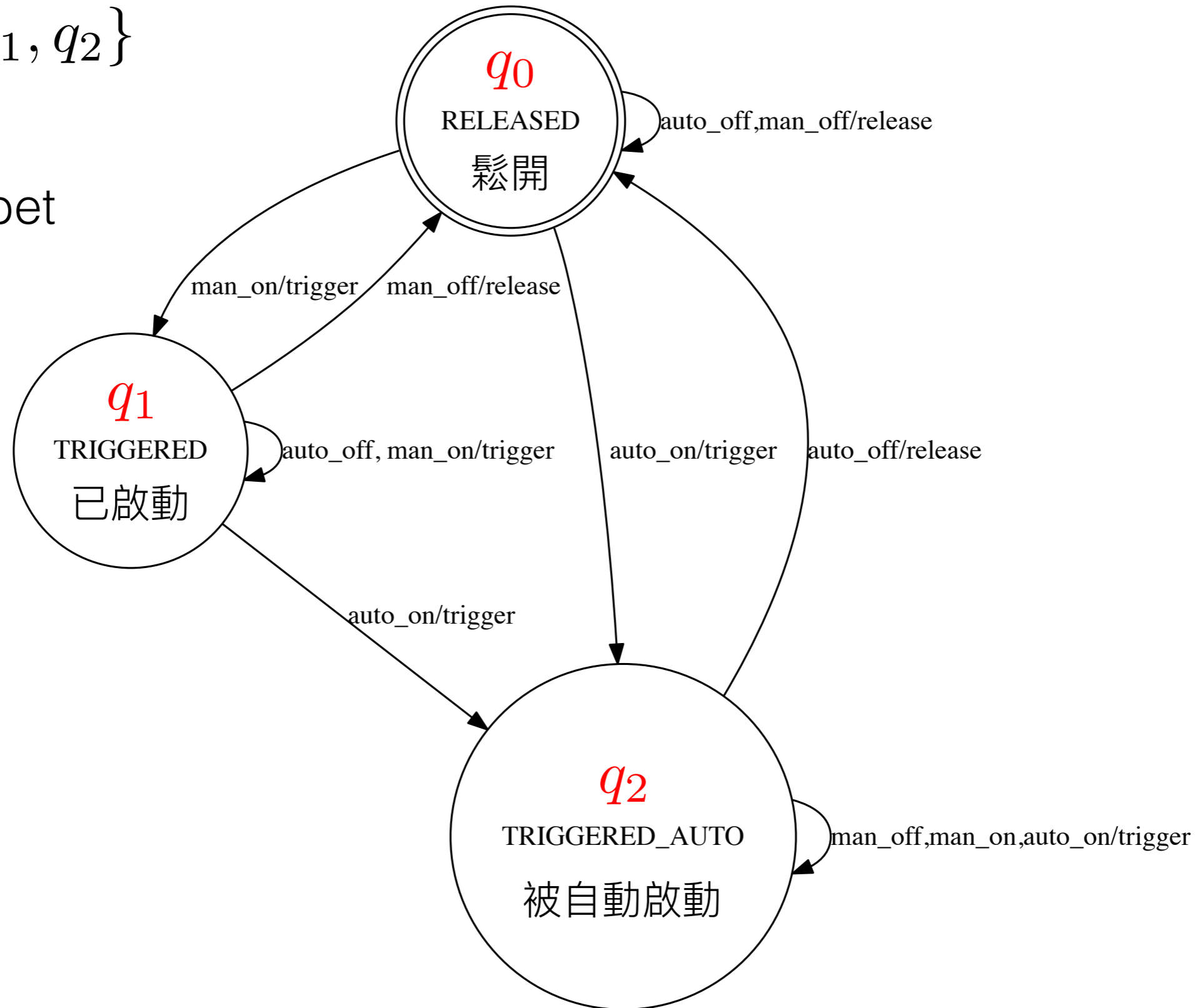
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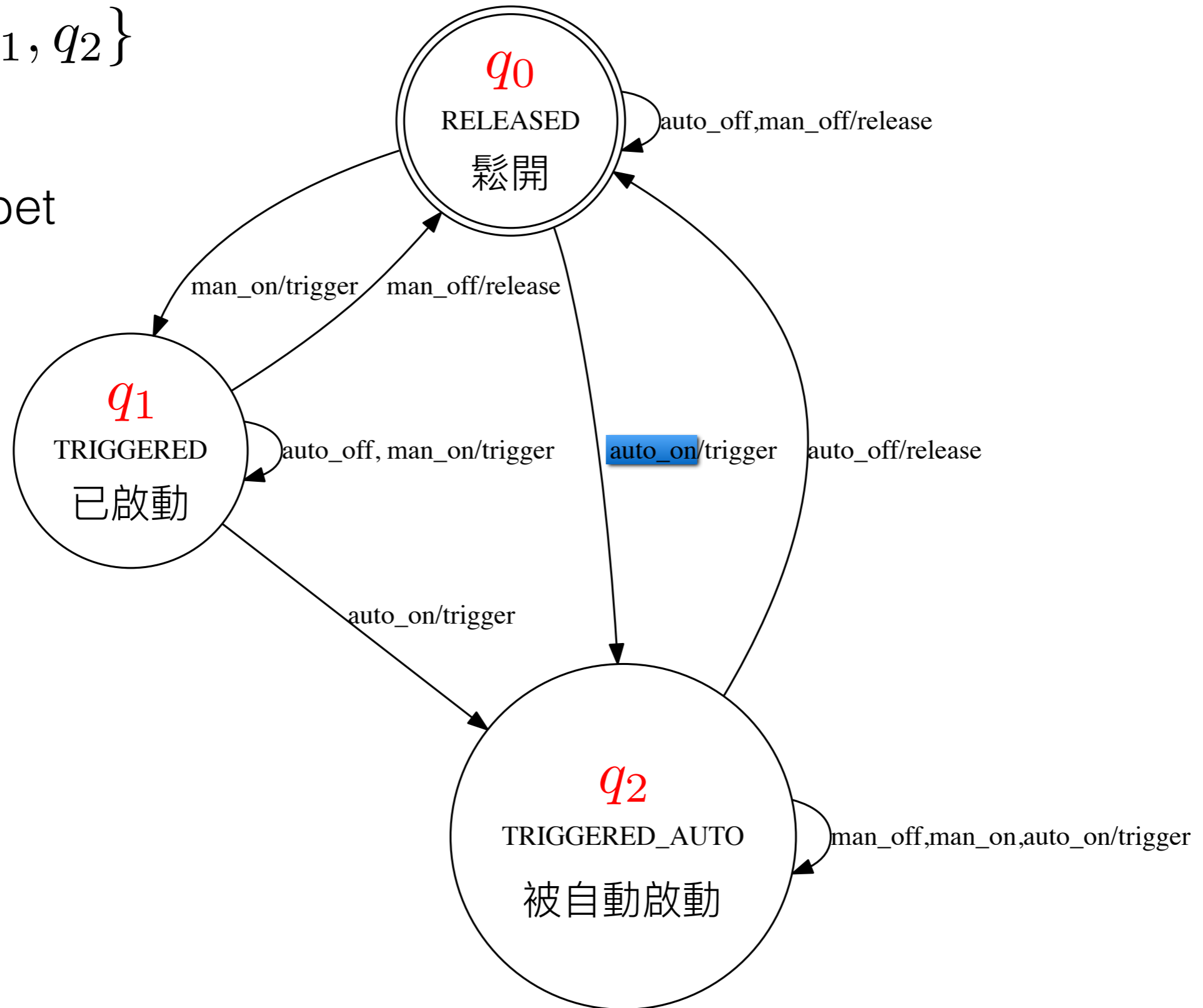
Input Alphabet



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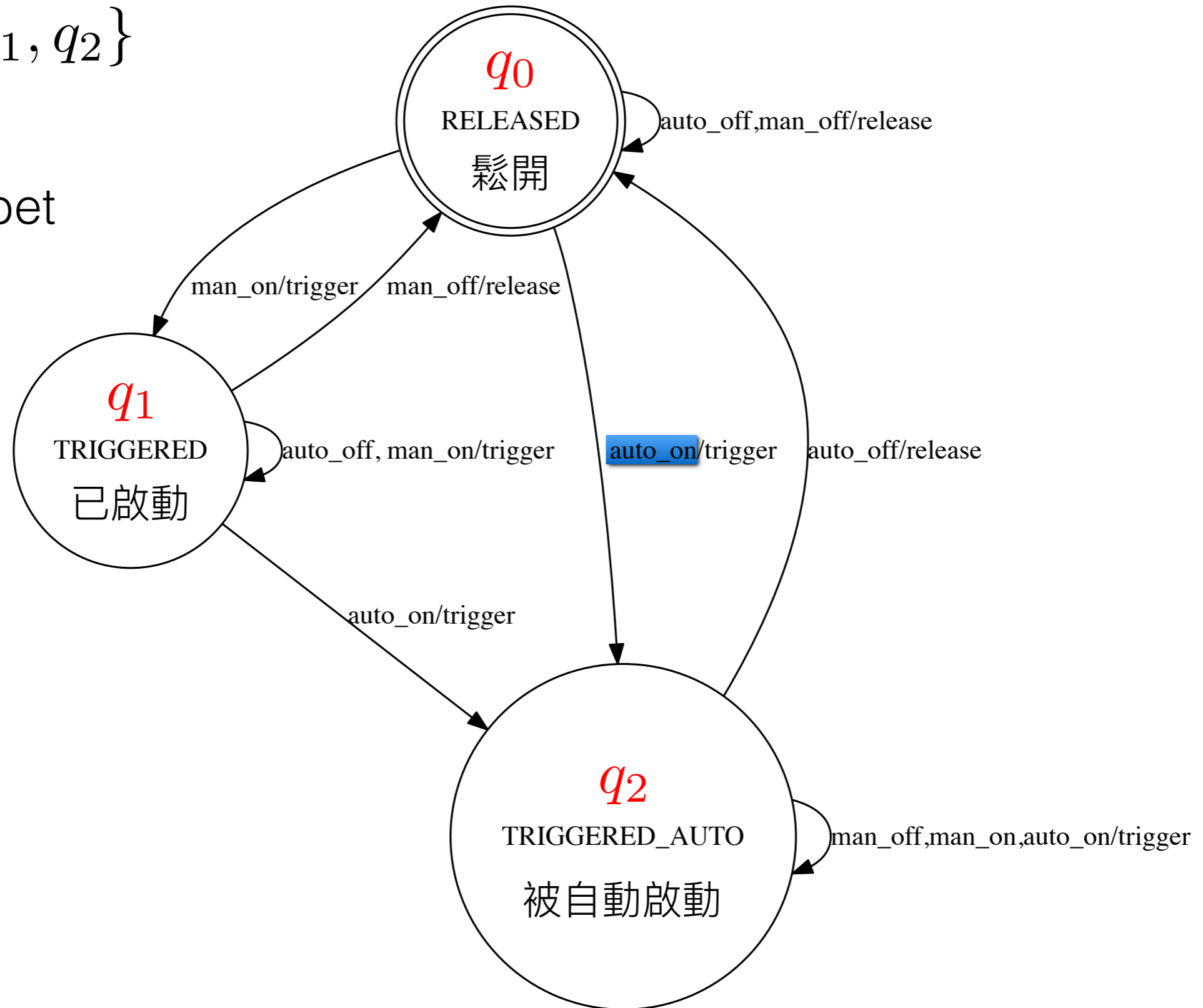


Finite State Machine modelling the behaviour of the brake controller

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Input Alphabet

man_on,
auto_on

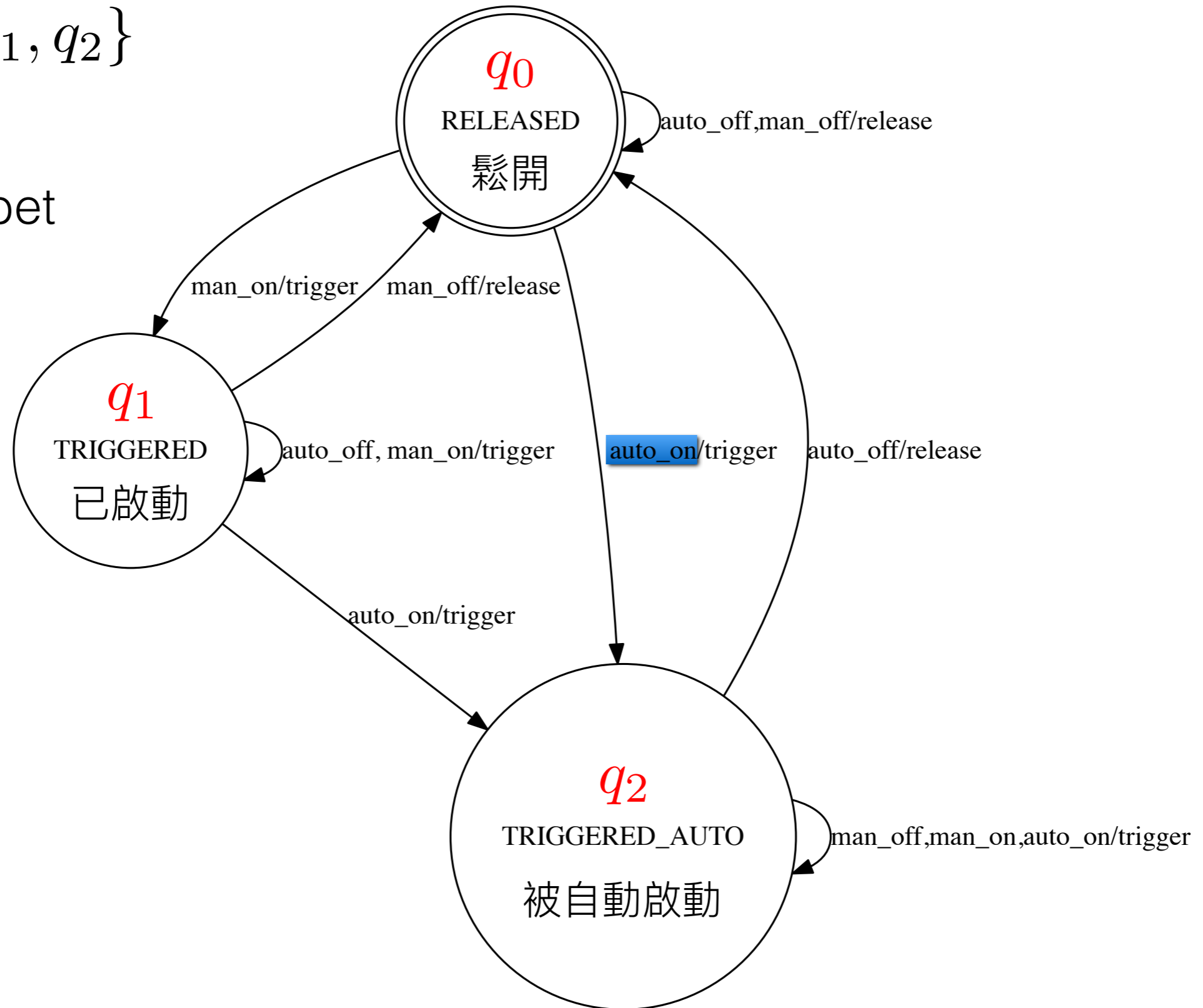


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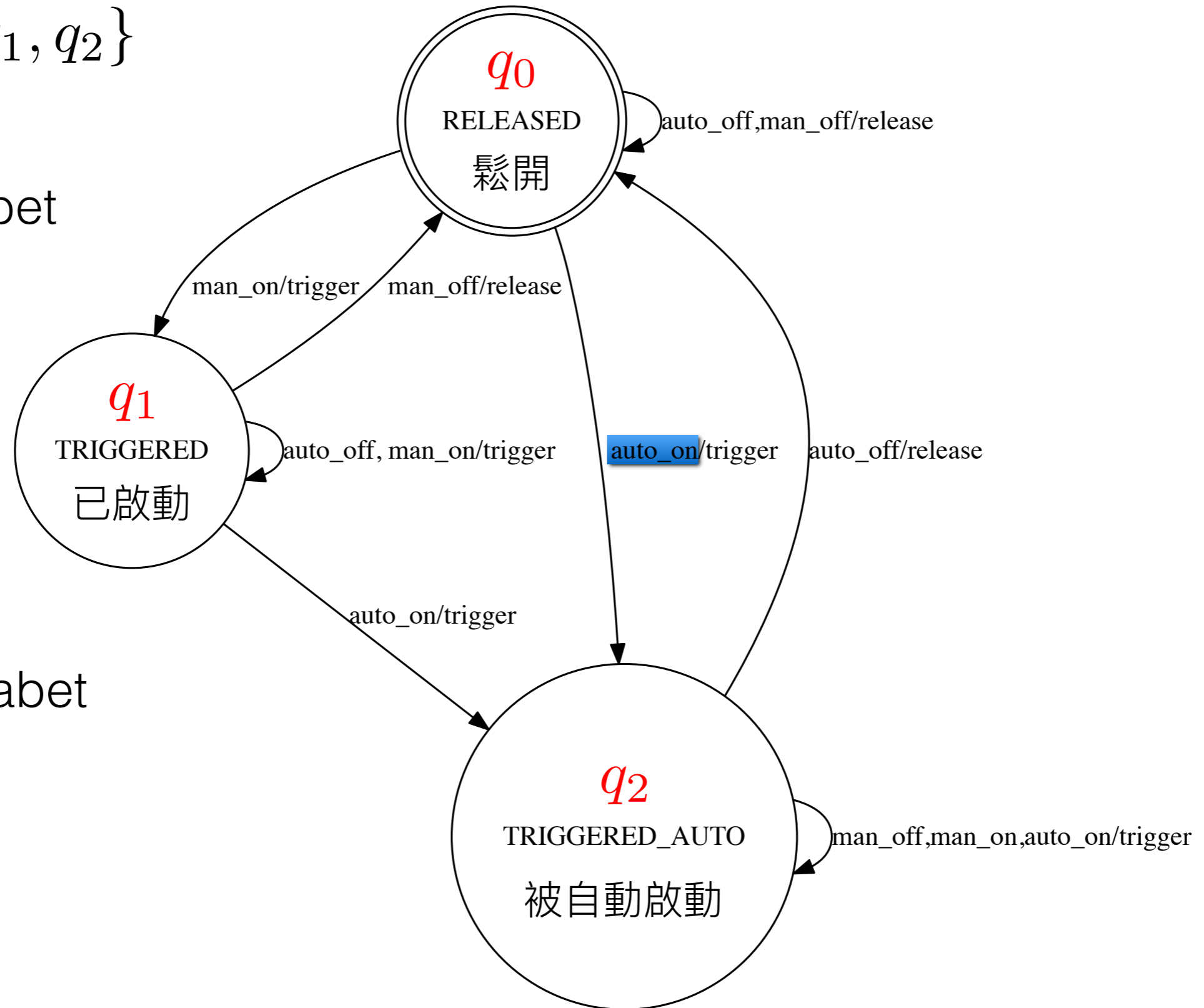


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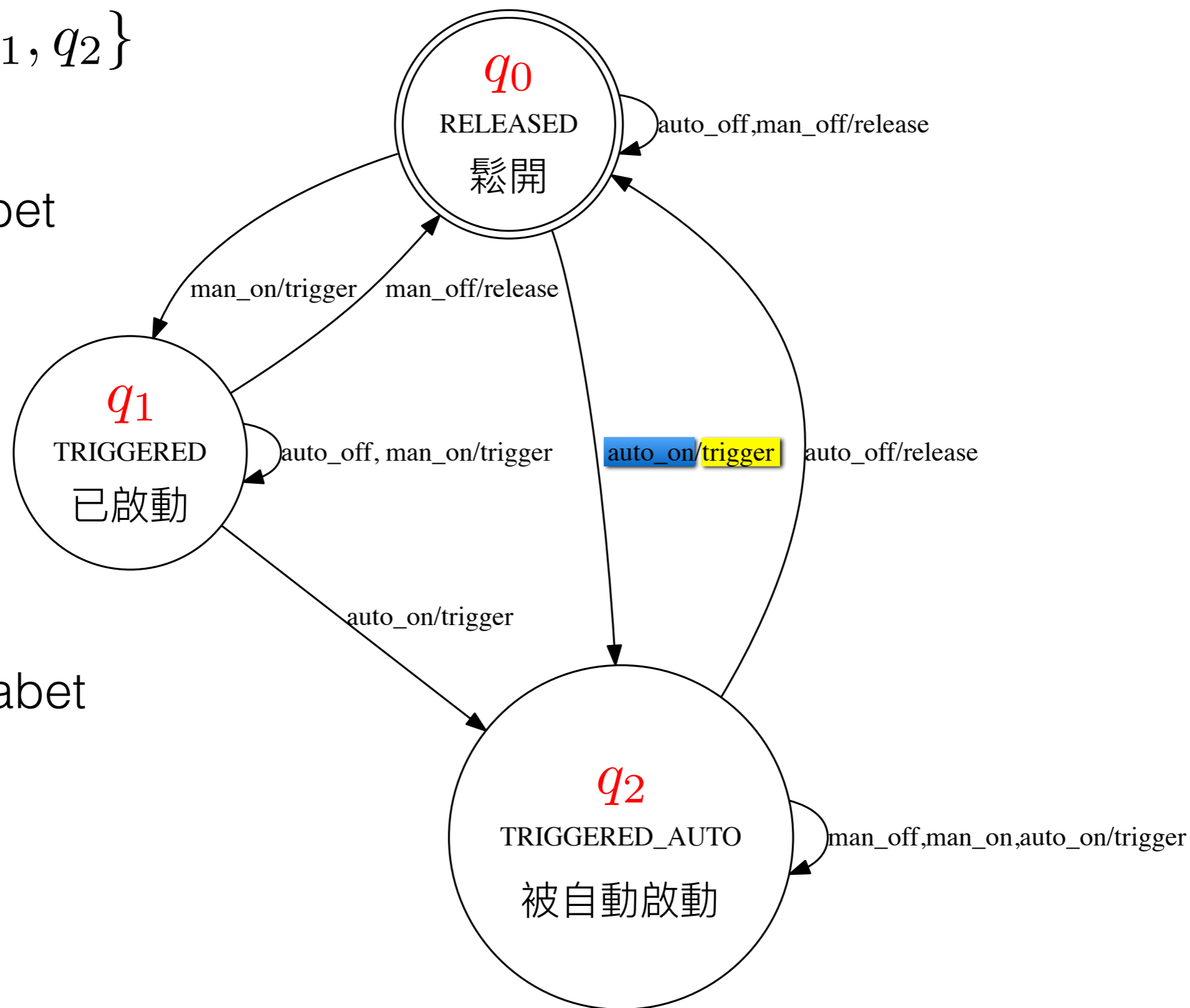


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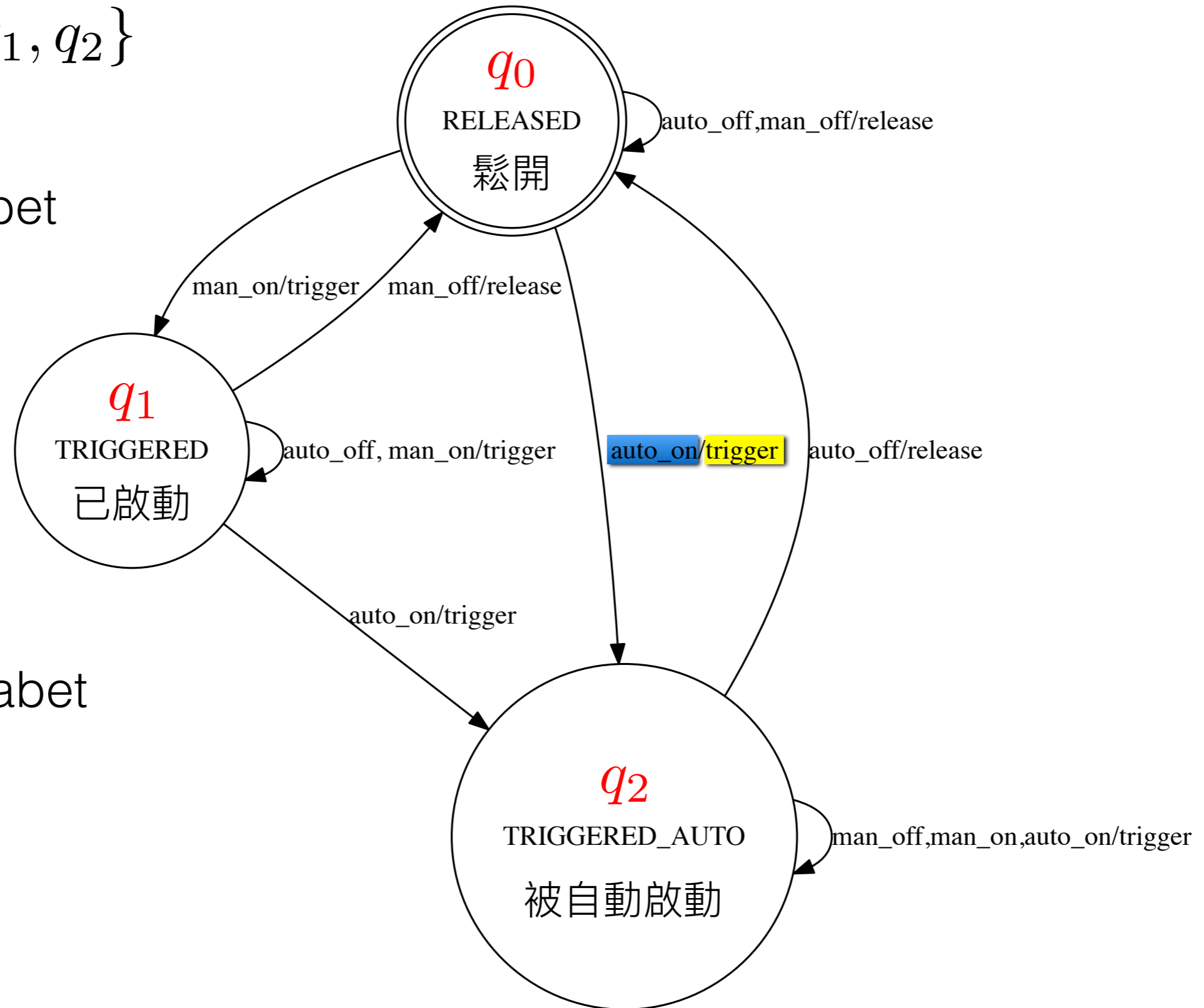


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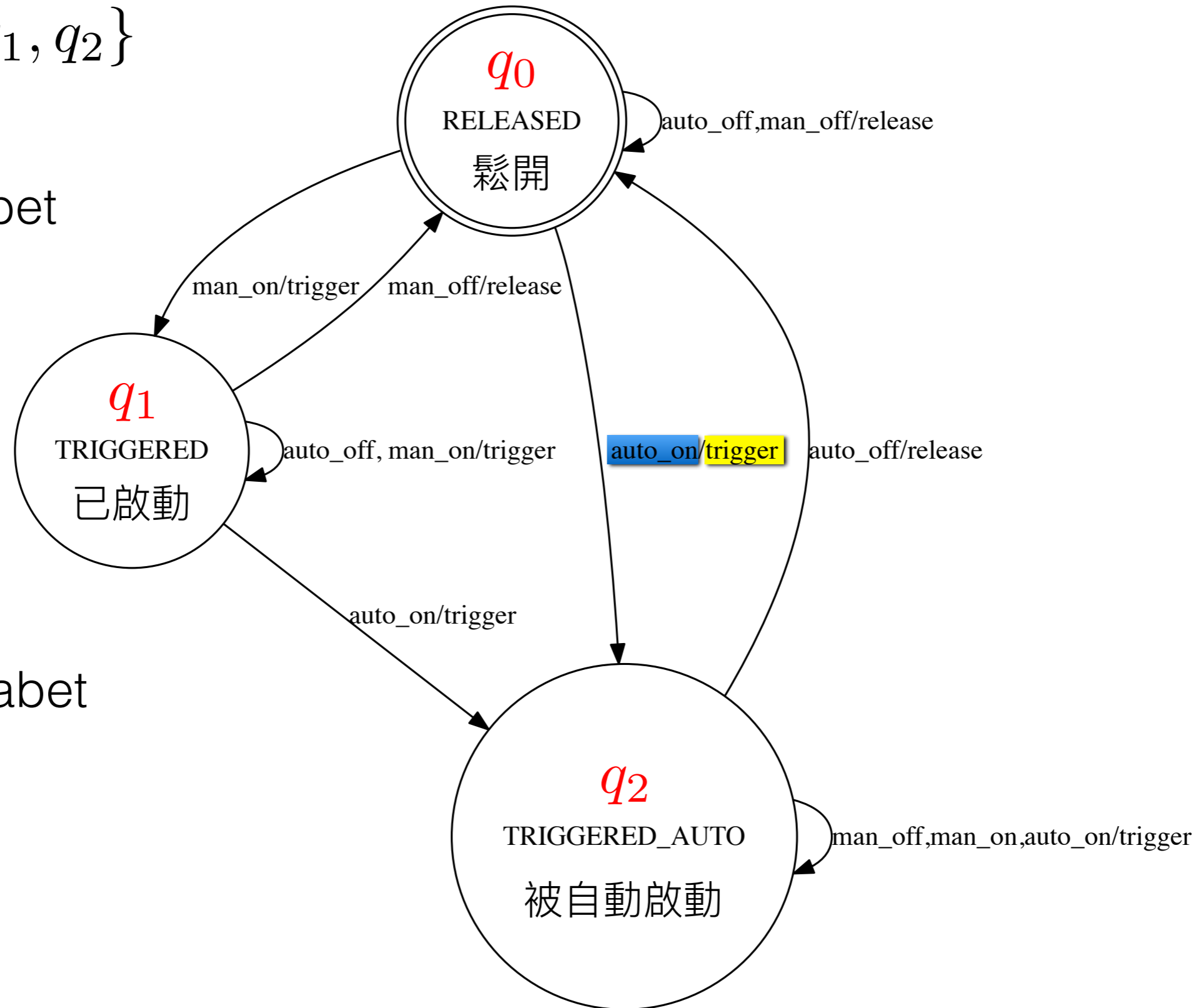
trigger

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Output Alphabet

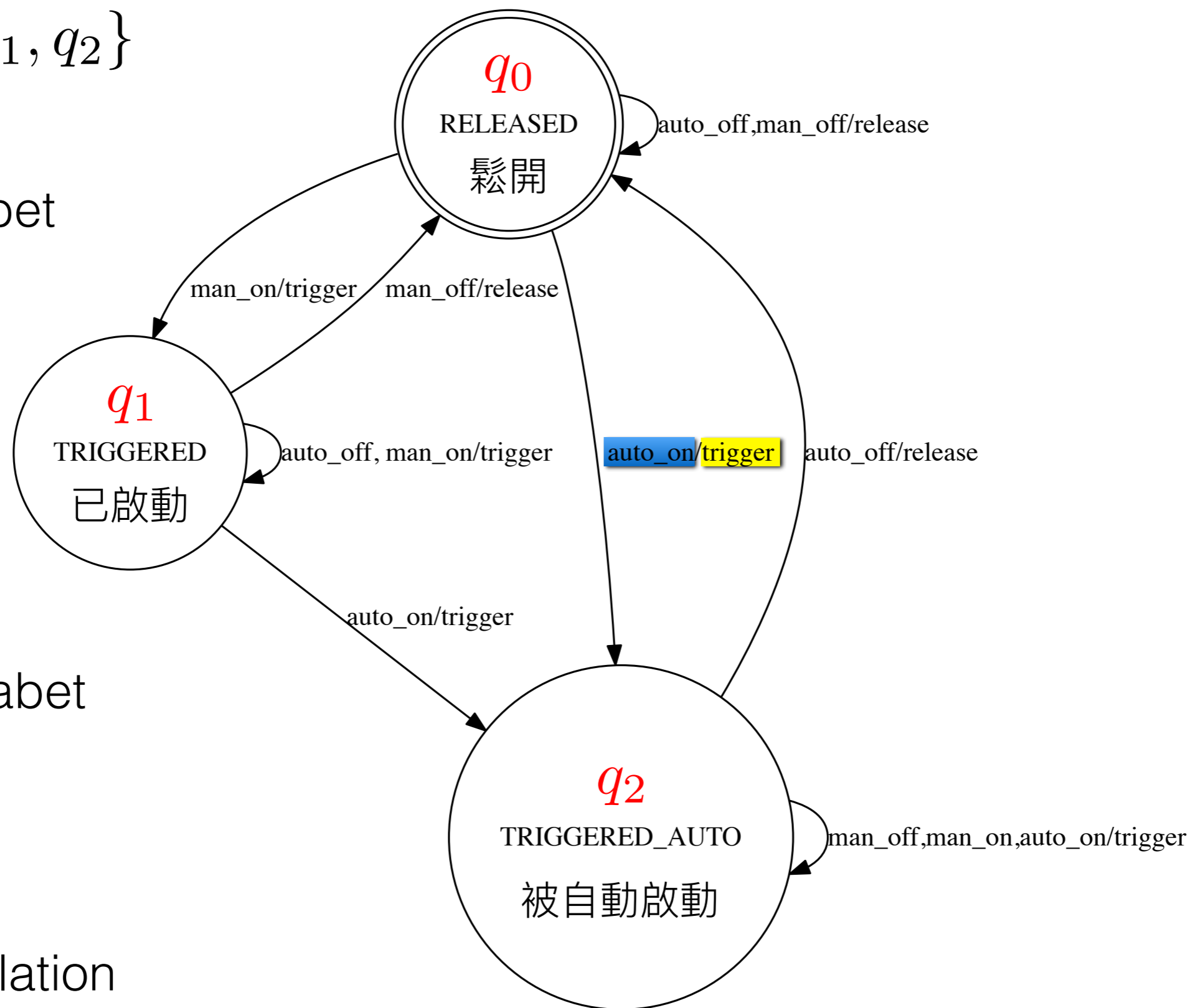
trigger
release

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Output Alphabet

trigger
release

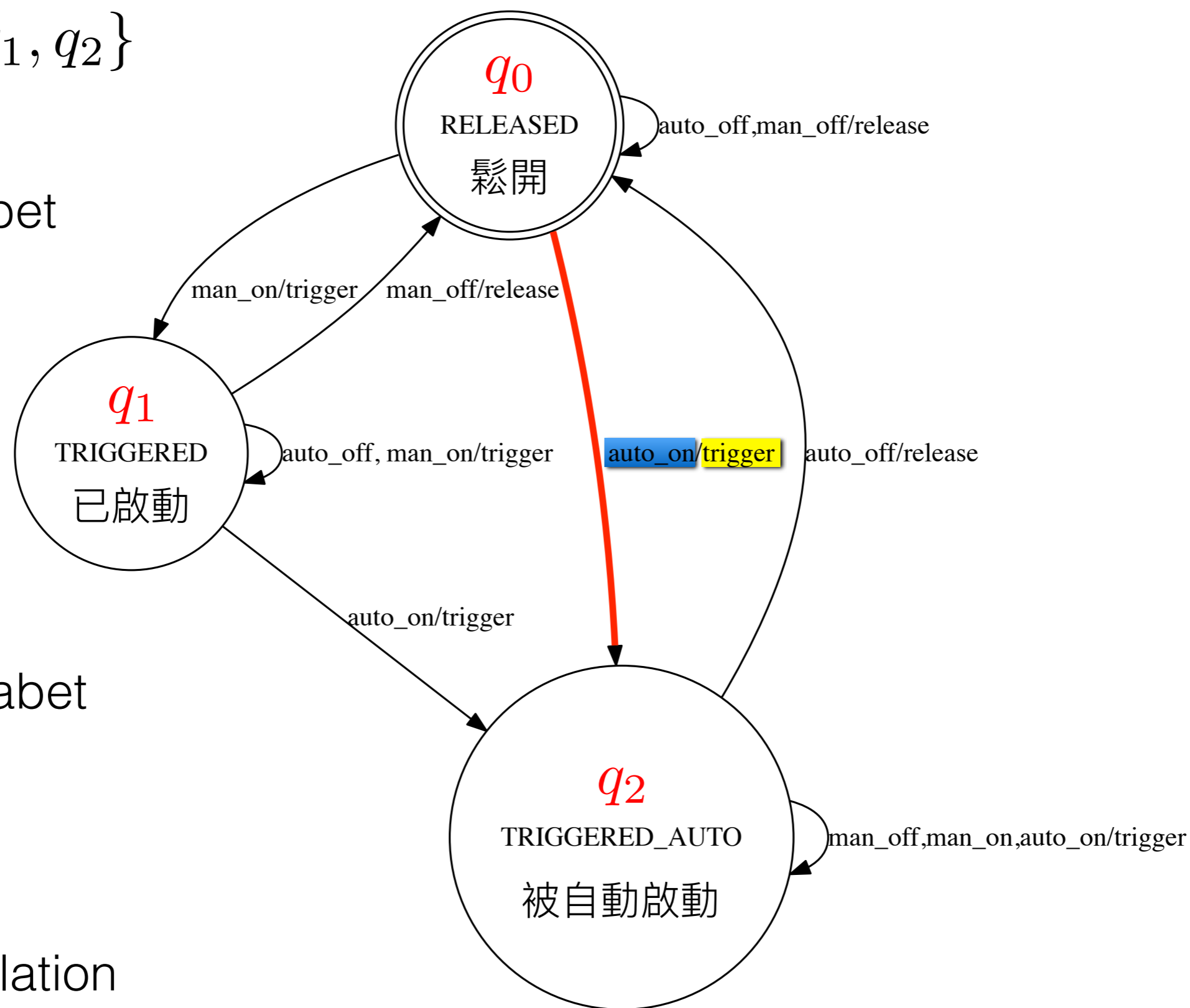
Transition Relation

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Output Alphabet

trigger
release

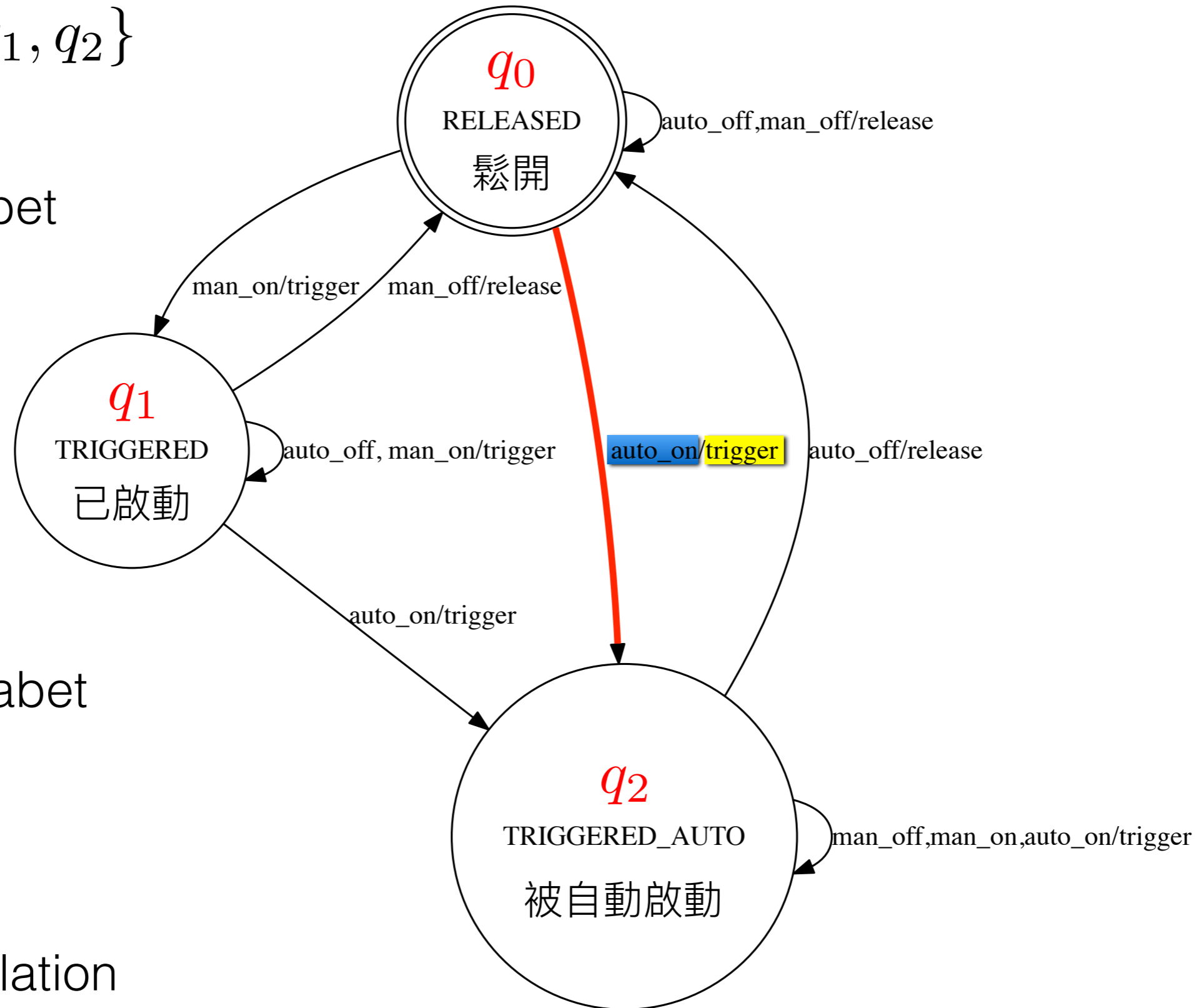
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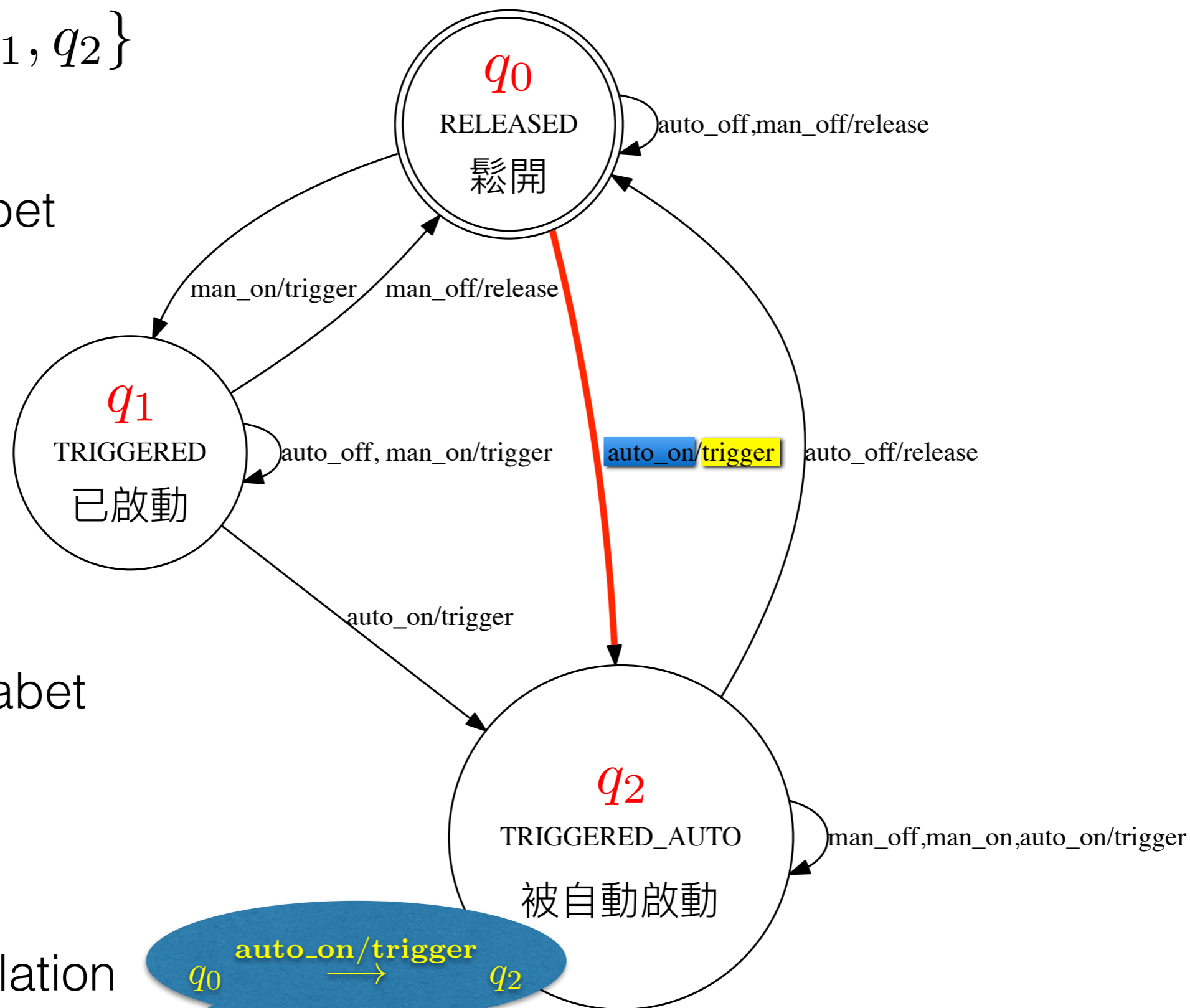
$(q_0, \text{auto_on}, \text{trigger}, q_2), \dots$

Finite State Machine modelling the behaviour of the brake controller

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Language

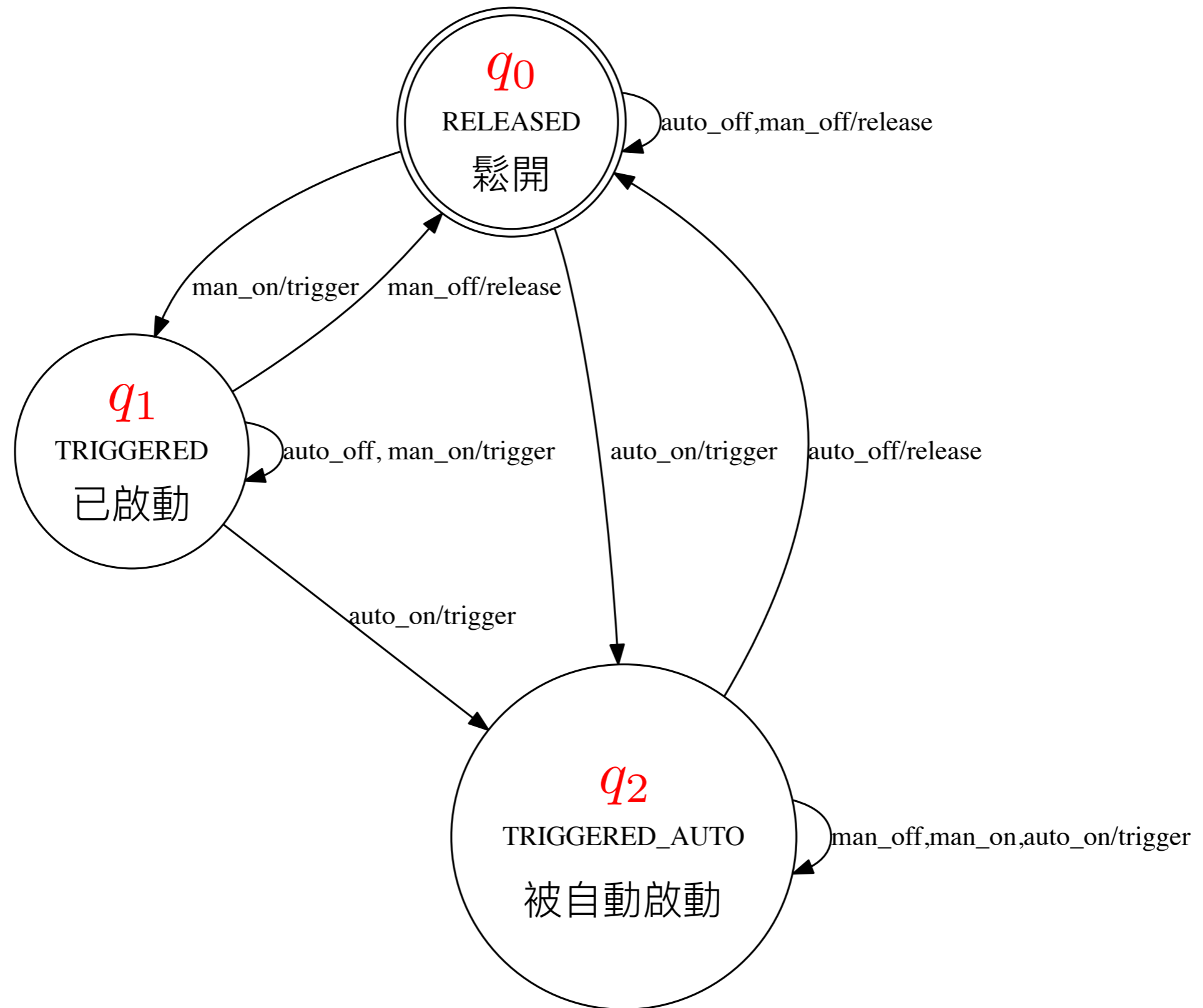
- $\Sigma := I \times O$ **alphabet** 字母表
- $\Sigma^* := \bigcup_{k \in \mathbf{N}_0} \{\sigma_1 \dots \sigma_k \mid \sigma_i \in \Sigma, \forall i = 1, \dots, k\}$ 有限字符序列集
- $\mathcal{L} \subseteq \Sigma^*$ **language over Σ** $:\Leftrightarrow$
 - $\varepsilon \in \mathcal{L}$ 空序列
 - \mathcal{L} is prefix-closed 前置序列

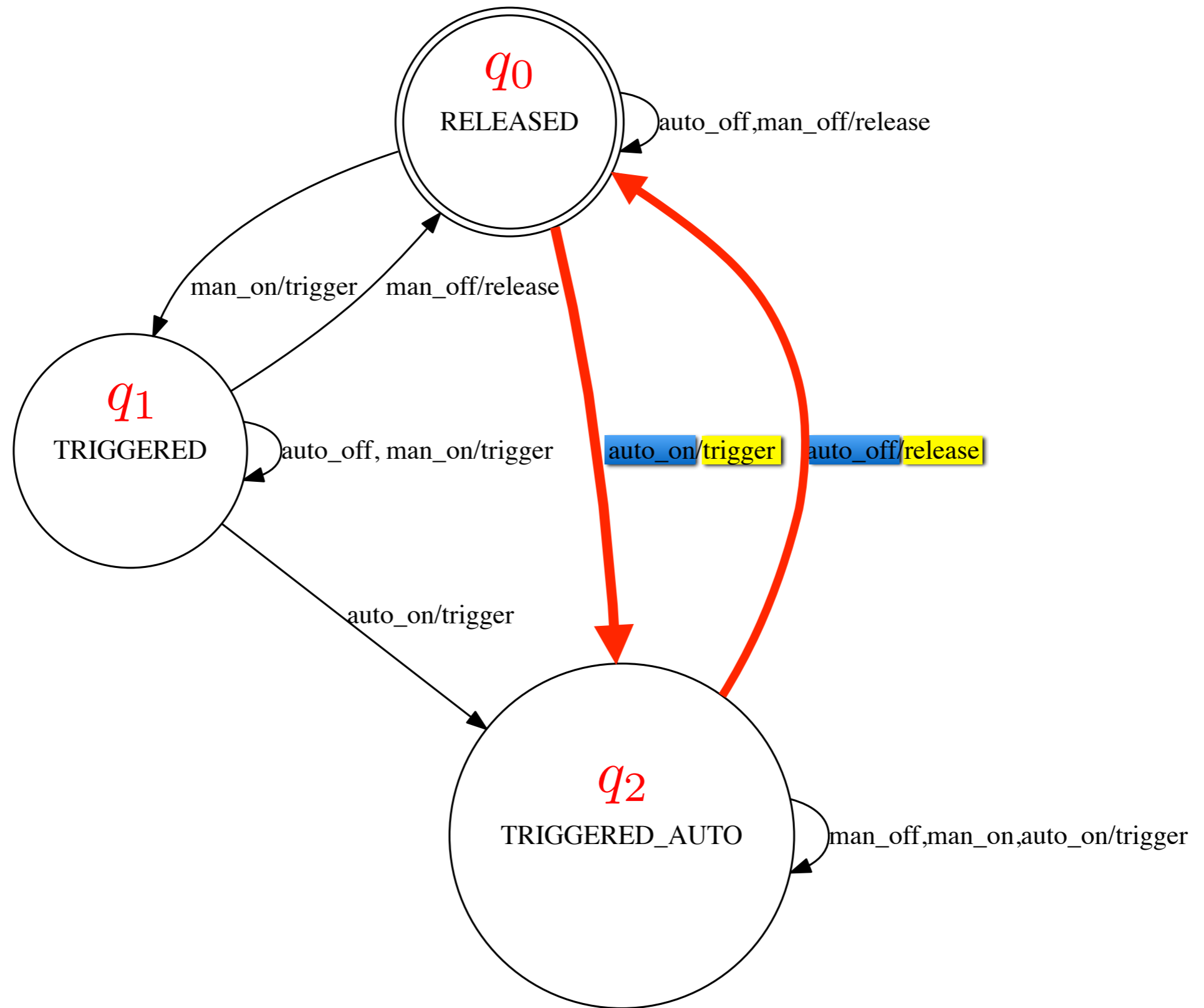
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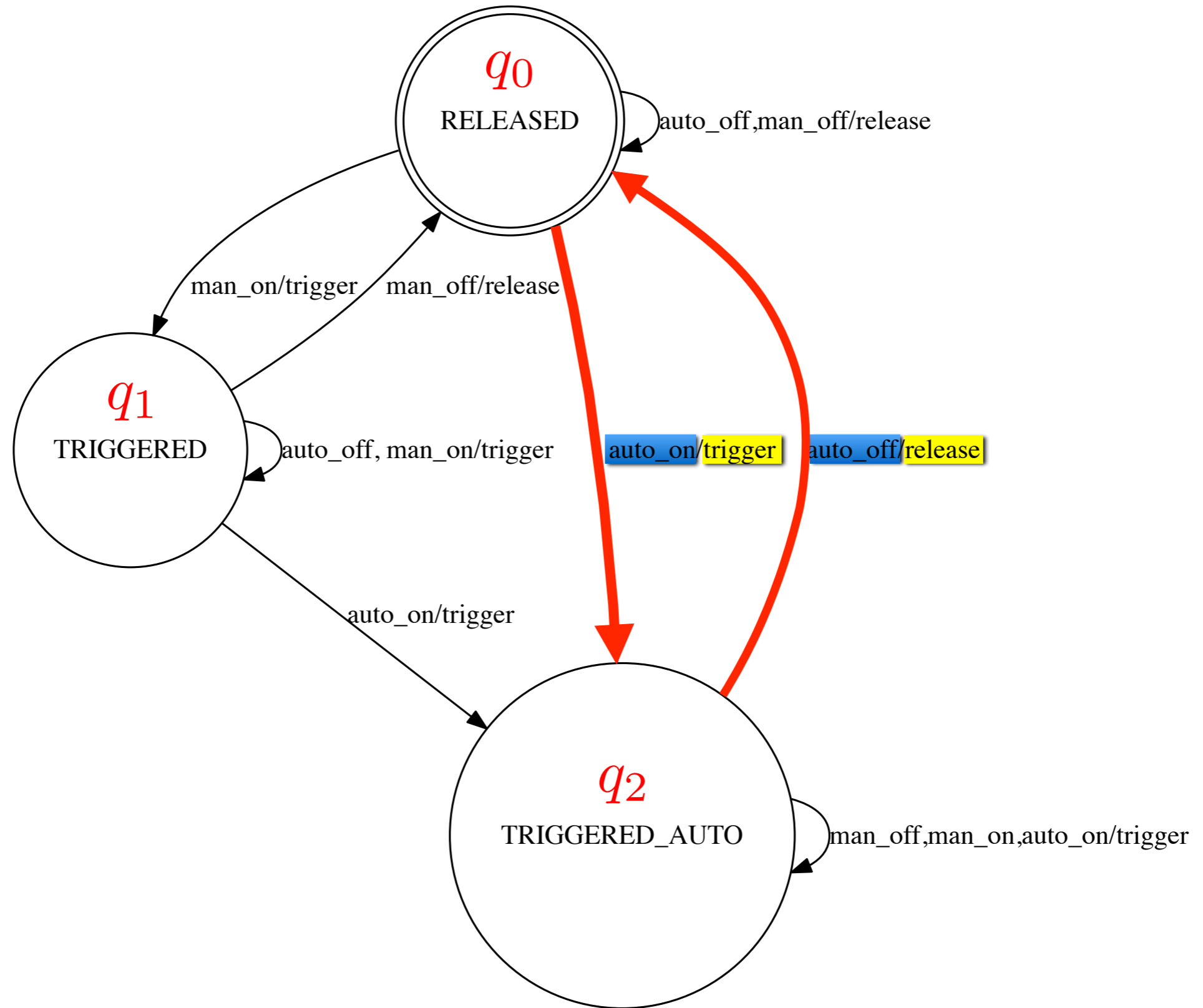
$$\forall \pi = \sigma_1 \dots \sigma_k \in \mathcal{L} : \sigma_1 \dots \sigma_i \in \mathcal{L}, \forall i \leq k$$

Finite State Machine modelling the behaviour of the brake controller



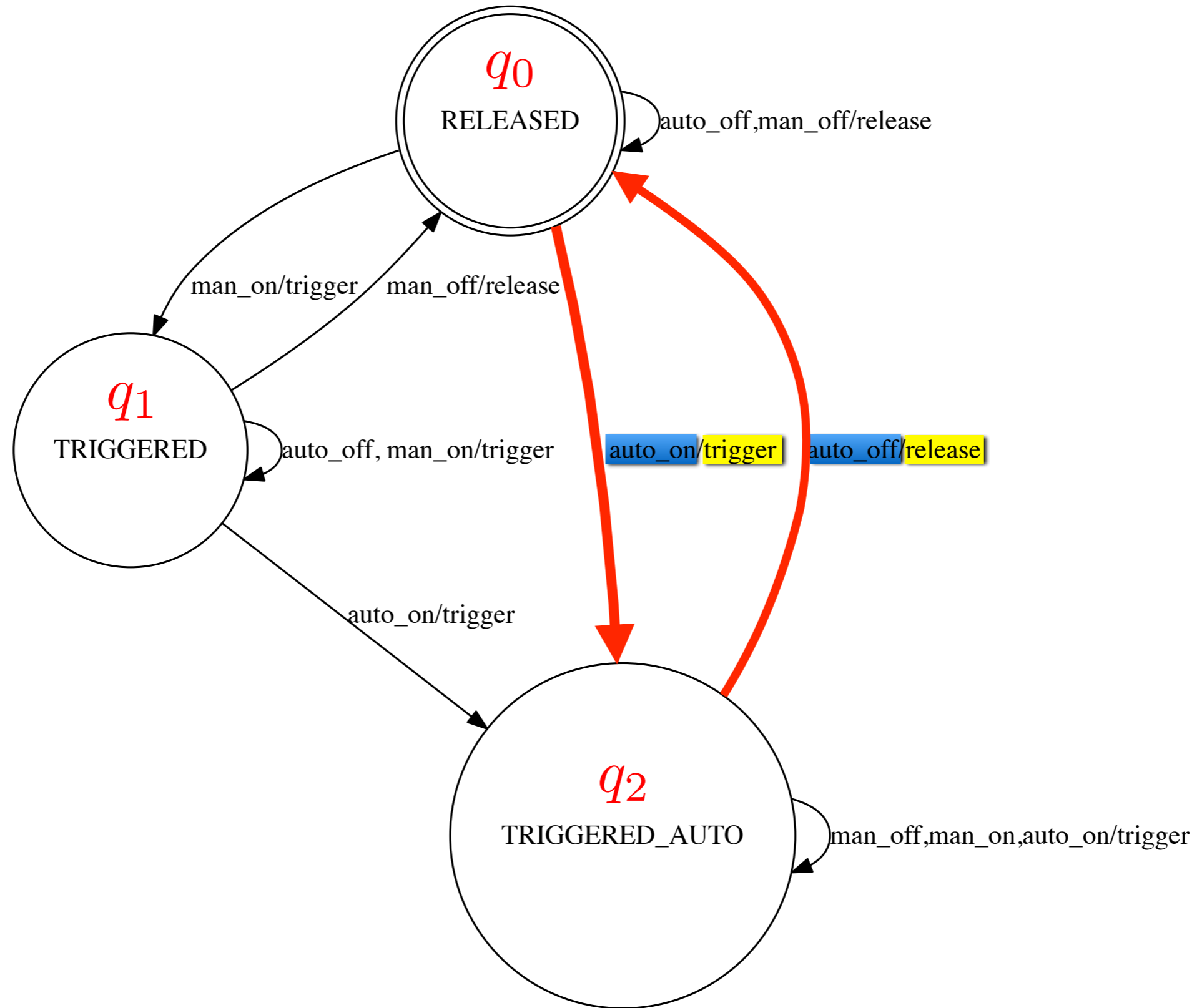


q_0 $\xrightarrow{\text{auto_on/trigger}}$ q_2 $\xrightarrow{\text{auto_off/release}}$ q_0



$q_0 \xrightarrow{\text{auto_on/trigger}} q_2 \xrightarrow{\text{auto_off/release}} q_0$

$(\text{auto_on, trigger}).(\text{auto_off, release}) \in L(q_0)$



Language of FSM

$$M = (Q, q_0, I, O, h)$$

- $L(q) := \{\pi \in \Sigma^* \mid \exists q' \in Q, q \xrightarrow{\pi} q'\}$ **language of q**
- $q \sim q' :\Leftrightarrow L(q) = L(q')$ **q q' are equivalent**
- $L(M) := L(q_0)$ **language of M**

Conformance Relations

$M = (Q, q_0, I, O, h)$, $M' = (Q', q'_0, I, O, h')$ two FSM.

- M and M' are **I/O equivalent** : $L(M') = L(M)$
- M' is an **I/O reduction** of M : $L(M') \subseteq L(M)$

FSM Properties

$$M = (Q, q_0, I, O, h)$$

- M is **input-complete** : 輸入完備性

$$\forall q \in Q \wedge x \in I, \exists y \in O \wedge q' \in Q : q \xrightarrow{x/y} q'$$

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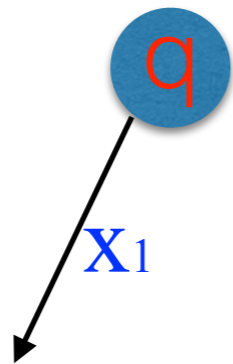
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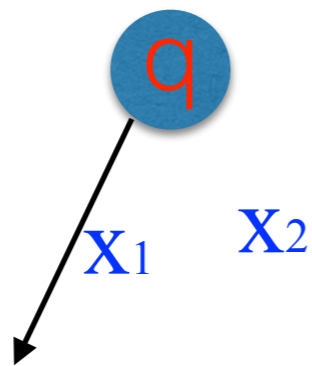


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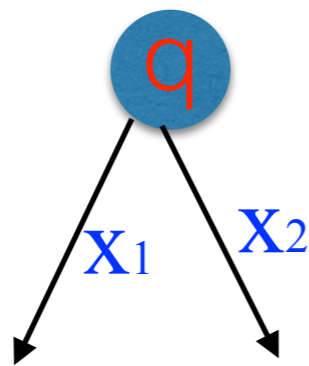


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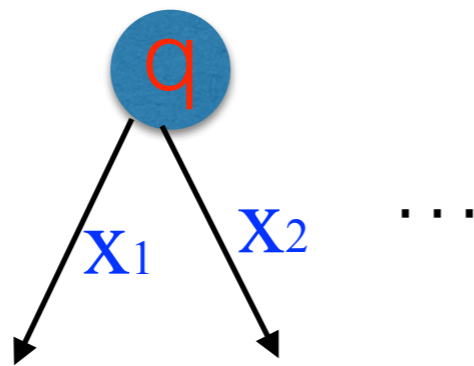


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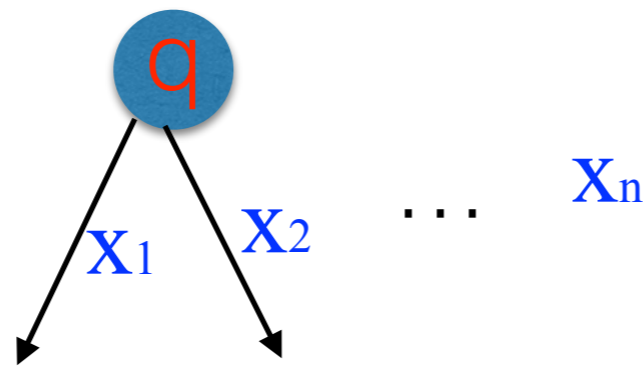


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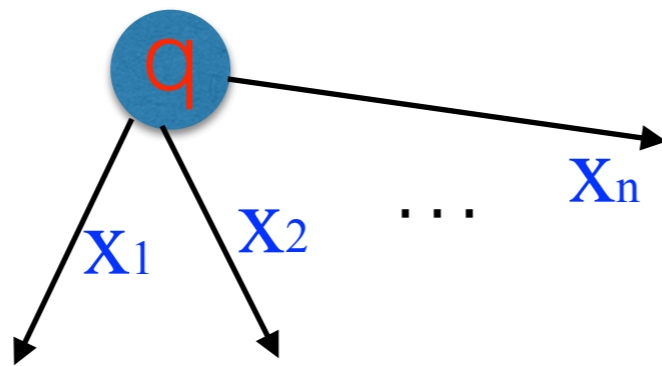


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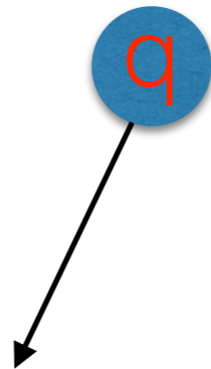
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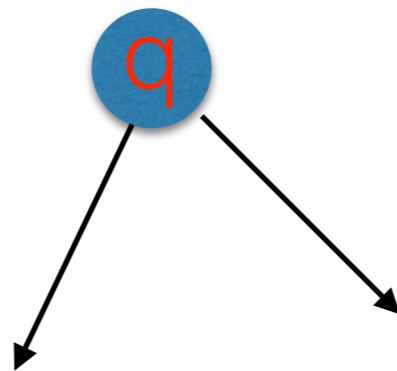
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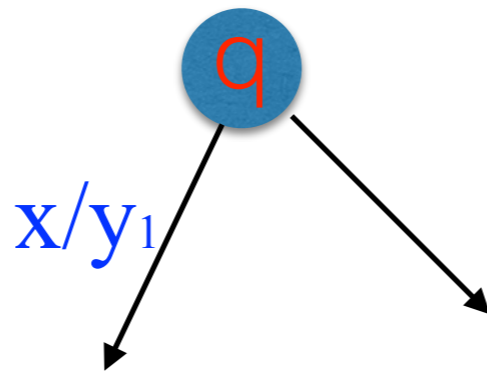
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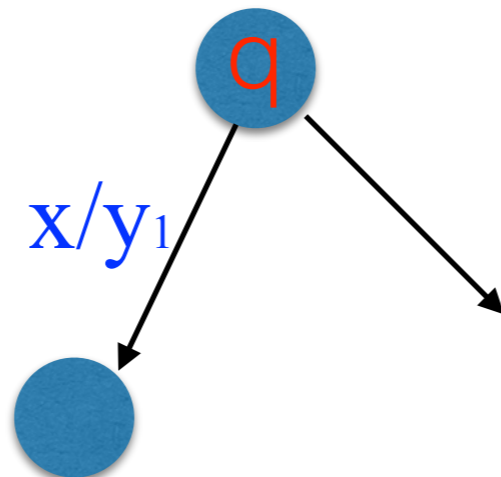
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$$\forall q \xrightarrow{x/y_1} q_1, q \xrightarrow{x/y_2} q_2 \in h \Rightarrow y_1 = y_2 \wedge q_1 = q_2$$



FSM Properties

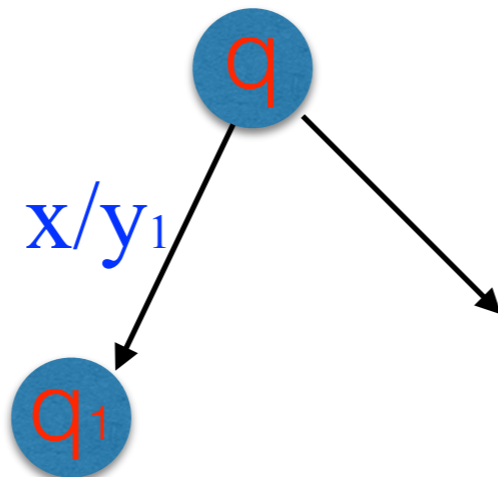
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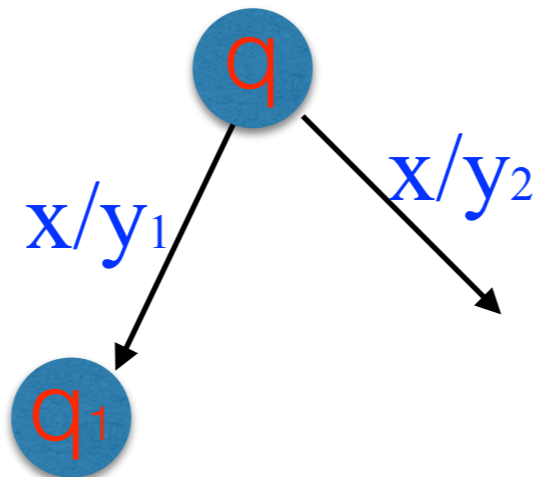
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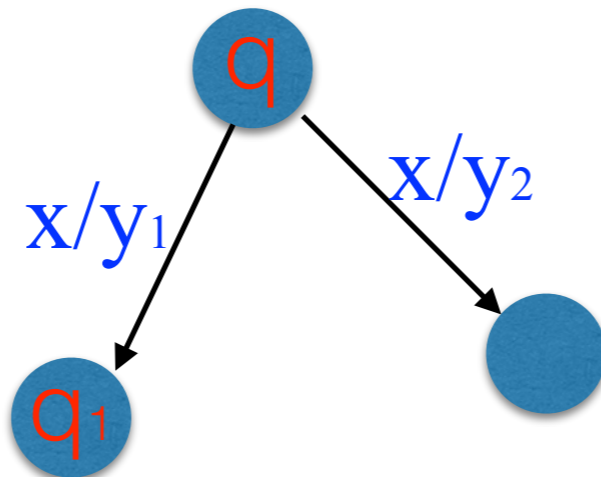
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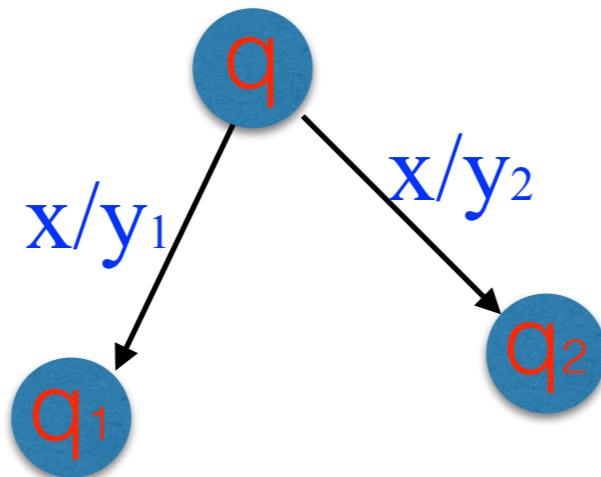
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FSM Properties

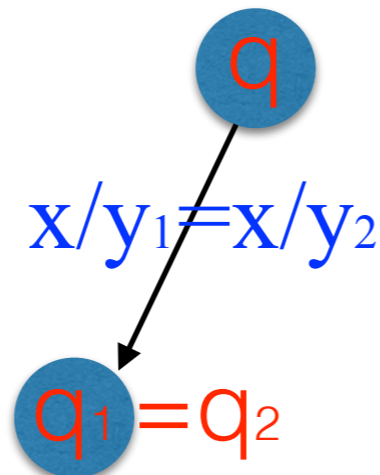
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FSM Properties

可觀察的

- M is **observable**, if

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極小系統

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可觀察的

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M is deterministic $\Rightarrow M$ is observable

極小系統

- M is **minimal**, if

$$- \forall q \in Q, \exists \pi \in L(q_0) : q_0 \xrightarrow{\pi} q$$

$$- \forall q_1 \neq q_2 \in Q \Rightarrow L(q_1) \neq L(q_2)$$

FSM Properties

可觀察的

- M is **observable**, if

$$\forall q \xrightarrow{x/y} q_1, q \xrightarrow{x/y} q_2 \in h \Rightarrow q_1 = q_2$$

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極小系統

- M is **minimal**, if

- $\forall q \in Q, \exists \pi \in L(q_0) : q_0 \xrightarrow{\pi} q$

initial connected

- $\forall q_1 \neq q_2 \in Q \Rightarrow L(q_1) \neq L(q_2)$

Signature Sig is a set of

- input-complete
- minimal
- observable

finite state machines over $\Sigma = I \times O$

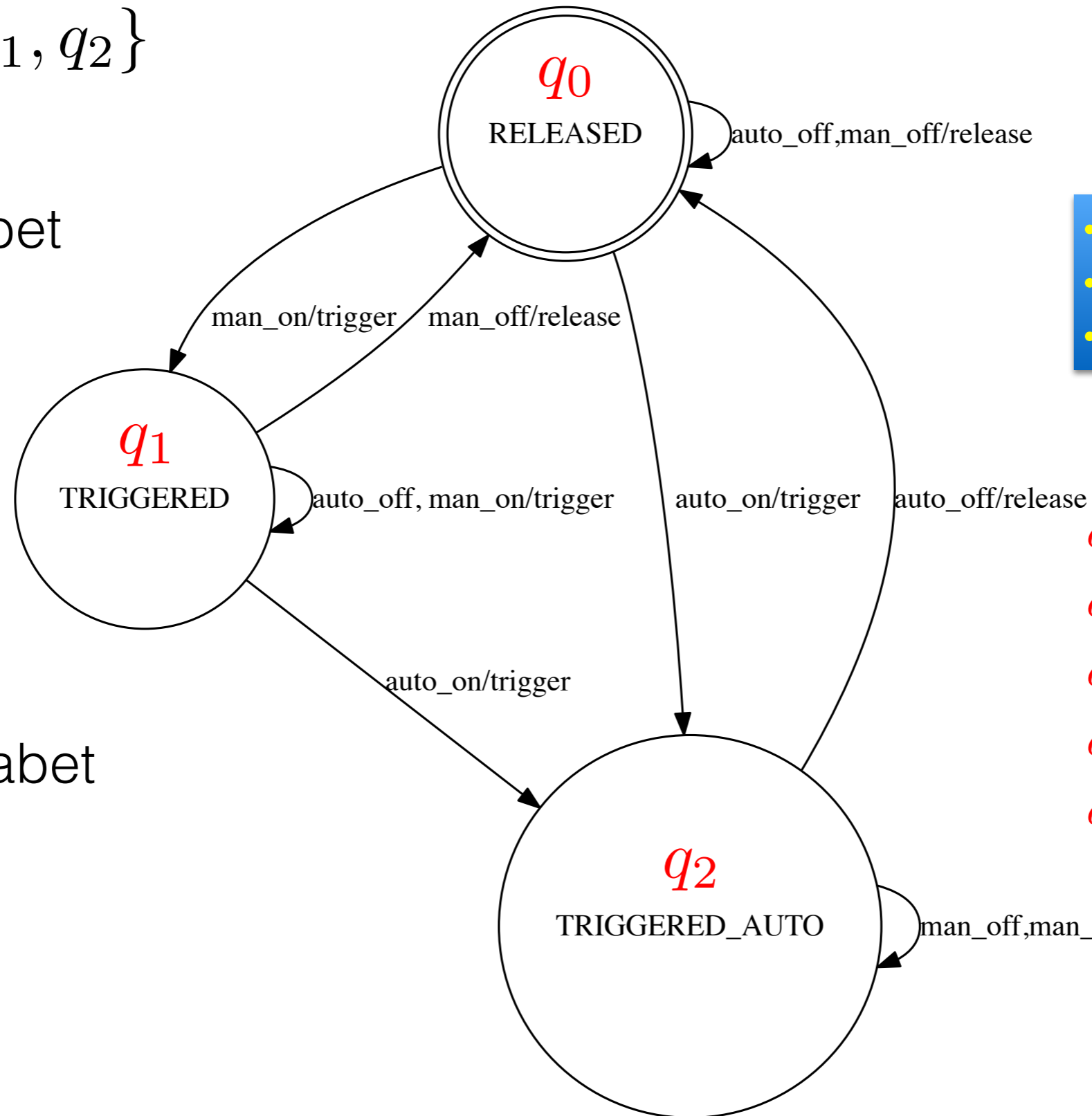
$$\forall M \text{ over } \Sigma, \exists M' \in \mathbf{Sig} : L(M) = L(M')$$

Finite State Machine modelling the behaviour of the brake controller

$$Q = \{q_0, q_1, q_2\}$$

Input Alphabet

man_on,
auto_on
man_off,
auto_off



- **deterministic**
- **input complete**
- **minimal**

Output Alphabet

trigger
release

q0 $\xrightarrow{\text{auto_off/release}}$ *q0*
q1 $\xrightarrow{\text{auto_off/trigger}}$ *q1*
q2 $\xrightarrow{\text{auto_off/release}}$ *q0*
q0 $\xrightarrow{\text{man_off/release}}$ *q0*
q2 $\xrightarrow{\text{man_off/trigger}}$ *q2*

$$M = (Q, q_0, I, O, h), M' = (Q', q'_0, I, O, h') \in \text{Sig}$$

$$L(M) = L(M') \Leftrightarrow M, M' \text{ **isomorphic**}$$

$$\exists f : M \rightarrow M':$$

- $Q \mapsto Q'$ bijection
- $q_0 \mapsto q'_0$
- $I \mapsto I$ identity map
- $O \mapsto O$ identity map
- $q_1 \xrightarrow{x/y} q_2 \in h \Leftrightarrow f(q_1) \xrightarrow{x/y} f(q_2) \in h'$

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Fault Models

$$\mathcal{F}(M, I, O, \leq, \mathcal{D})$$

- $M \in Sig$, **reference model** 參照模型
- $\leq \subseteq Sig \times Sig$, **conformance relation** 形變關係
(I/O equivalence or I/O reduction)
- $\mathcal{D} \subseteq Sig$, **fault domain** 錯誤域

Test Cases

測試範例

Test case of deterministic FSM:

I/O sequence $\pi = x_1/y_1 \dots x_k/y_k \in \Sigma^*$

- M **passes** π , if $\pi \in L(M')$
- M **fails** π , if $\pi \notin L(M')$

Test Suite

測試範例組

test suite TS : a collection of test cases.

- M **passes** TS , if
 $\forall \pi \in TS, M$ passes π .
- M **fails** TS , if
 $\exists \pi \in TS, M$ fails π .

Complete Test Suites

完備測試範例組

$\mathcal{F}(M, I, O, \leq, \mathcal{D})$, fault model

TS, test suite

可靠性

- **Soundness**: $\forall M' \in \mathcal{D} : M' \leq M \Rightarrow M' \underline{\text{pass}} \mathbf{TS}$

全面性

- **Exhaustiveness**: $\forall M' \in \mathcal{D} : M' \underline{\text{pass}} \mathbf{TS} \Rightarrow M' \leq M$

完備性

- **Completeness**: Soundness + Exhaustiveness

$$\forall M' \in \mathcal{D} : M' \leq M \Leftrightarrow M' \underline{\text{pass}} \mathbf{TS}$$

Testing Theories

- **Deterministic FSM:**
- **T-Method**
- **Product Automata**
- **W-Method, Wp-Method**
- **Nondeterministic FSM:**
- **W-Method, Wp-Method**
- **Adaptive Testing**

Signature Sig is the set of

- input-complete
- minimal
- **deterministic**

finite state machines over $\Sigma = I \times O$

T-Method

$\mathcal{F}(M, I, O, \leq, \mathcal{D}_O)$, fault model

$M = (Q, q_0, I, O, h)$, $\forall M' \in \mathcal{D}_O \subseteq \text{Sig}$:

- $M' = (Q, q_0, I, O, h')$
- $\exists f : h \rightarrow h'$ bijective,
 $(q_1 \xrightarrow{x/y} q_2) \mapsto (q_1 \xrightarrow{x/y'} q_2)$

State Cover

State cover V of $M = (Q, q_0, I, O, h)$

- $V \subseteq L(M)$

- $\varepsilon \in V$

- $\forall q \in Q, \exists \pi \in V : q_0 \xrightarrow{\pi} q.$

$q_0\text{-after-}\pi = q$

State Cover **exists? finite?**

State cover V of $M = (Q, q_0, I, O, h)$

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$q_0\text{-after-}\pi = q$

Lemma

Let $M = (Q, q_0, I, O, h) \in \text{Sig}$ and $\varepsilon \in U \subseteq L(M)$.

Then U is a state cover of M or

$$\{q_0\text{-after-}\pi.\sigma \mid \pi \in U \wedge \sigma \in \Sigma \wedge \pi.\sigma \in L(M)\} \\ \not\subseteq \{q_0\text{-after-}\pi \mid \pi \in U\}.$$

Transition Cover

Transition cover P of $M = (Q, q_0, I, O, h)$

- $P \subset L(M)$
- $\varepsilon \in P$
- $\forall q \in Q, \sigma = x/y \in L(q), \exists \pi \in P : q_0 \xrightarrow{\pi} q \wedge \pi.\sigma \in P$

Transition Cover



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Transition Cover

Concatenation

For any $A, B \neq \emptyset \subseteq \Sigma^*$.

$A.B := \{\pi.\iota \mid \pi \in A, \iota \in B\}$

Example: $\Sigma = \{a, b, c\}$

$A = \{\varepsilon\}, B = \{a.b\}, C = \{a, c\}$

$A.B = \{\varepsilon.a.b\} = \{a.b\} = B$

$B.C = \{a.b.a, a.b.c\}$

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V is a state cover

$$\begin{aligned} \Rightarrow V \oplus (\{\varepsilon\} \cup \Sigma) &= (V.(\{\varepsilon\} \cup \Sigma)) \cap L(M) \\ &= V \cup \{\pi.\sigma \in L(M) \mid \pi \in V, \sigma \in \Sigma\} \end{aligned}$$

is a transition cover

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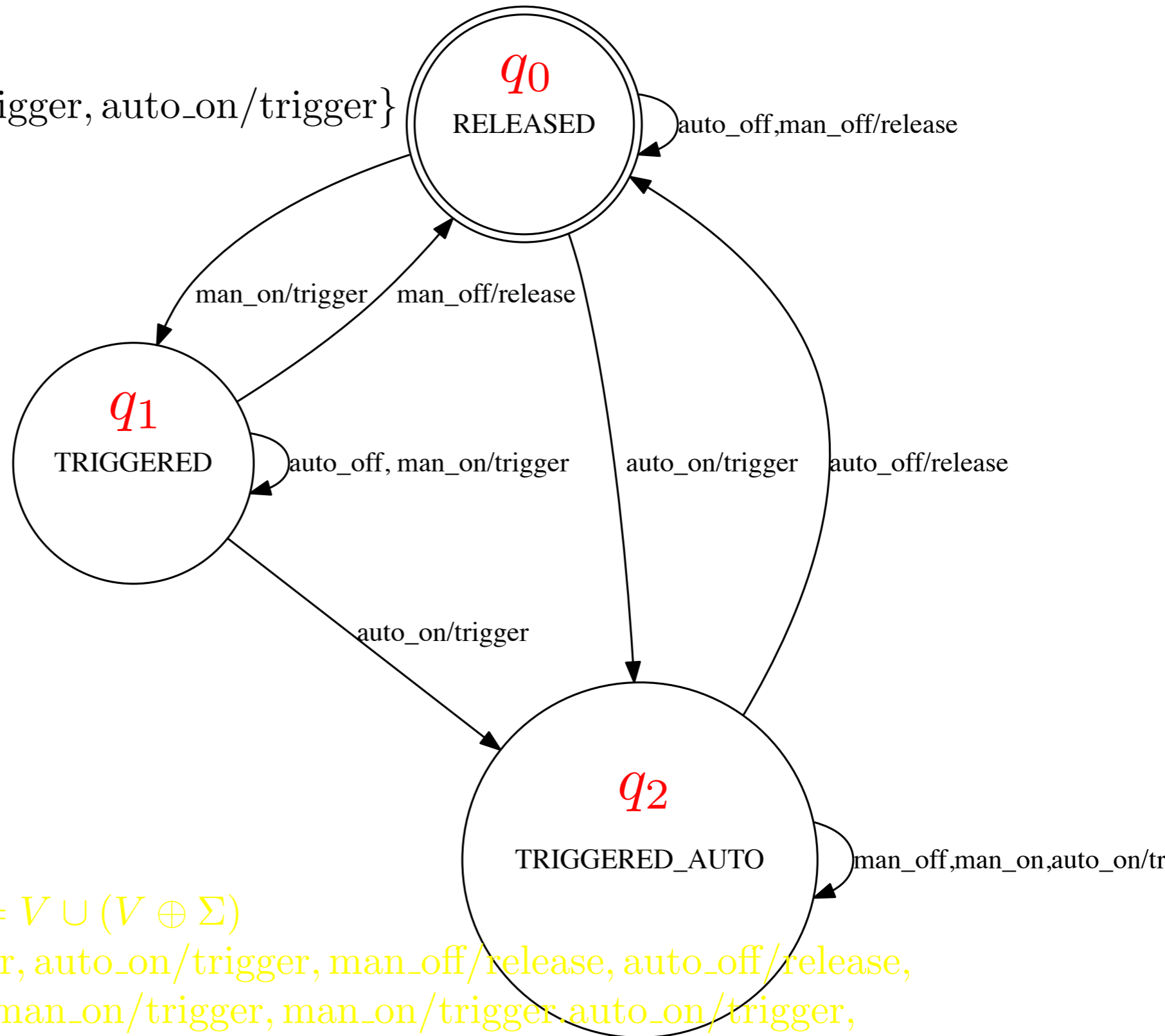
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is a transition cover

Finite State Machine modelling the behaviour of the brake controller

- state cover $V = \{\varepsilon, \text{man_on}/\text{trigger}, \text{auto_on}/\text{trigger}\}$

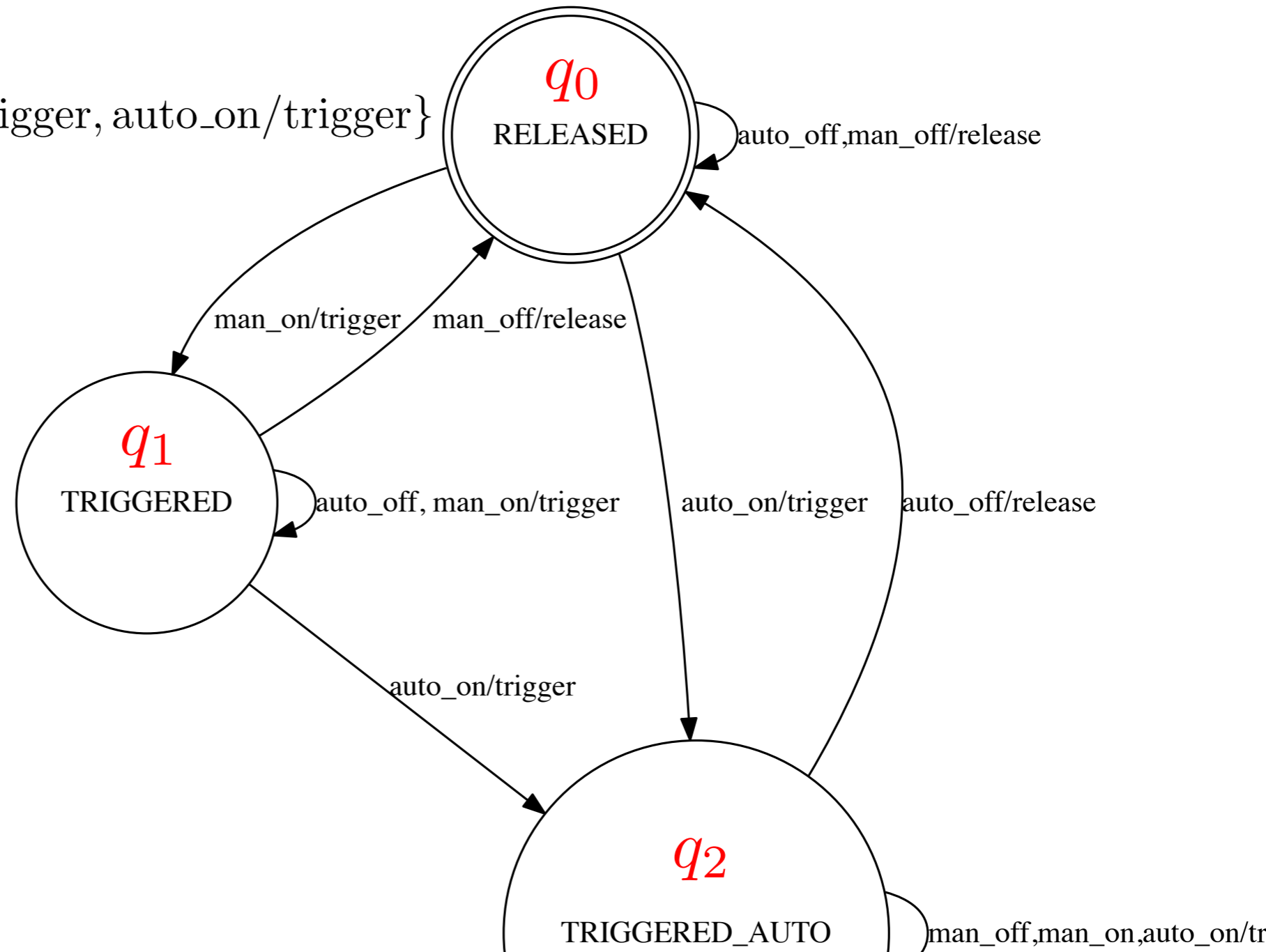


- transition cover $P = V \cup (V \oplus \Sigma)$

$= \{\varepsilon, \text{man_on}/\text{trigger}, \text{auto_on}/\text{trigger}, \text{man_off}/\text{release}, \text{auto_off}/\text{release}, \text{man_on}/\text{trigger}.\text{man_on}/\text{trigger}, \text{man_on}/\text{trigger}.\text{auto_on}/\text{trigger}, \text{man_on}/\text{trigger}.\text{man_off}/\text{release}, \text{man_on}/\text{trigger}.\text{auto_off}/\text{trigger}, \text{auto_on}/\text{trigger}.\text{man_on}/\text{trigger}, \text{auto_on}/\text{trigger}.\text{auto_on}/\text{trigger}, \text{auto_on}/\text{trigger}.\text{man_off}/\text{trigger}, \text{auto_on}/\text{trigger}.\text{auto_off}/\text{release}\}$

Finite State Machine modelling the behaviour of the brake controller

- state cover $V = \{\varepsilon, \text{man_on}/\text{trigger}, \text{auto_on}/\text{trigger}\}$



- transition cover $P = V \cup (V \oplus \Sigma)$
 $= \{\varepsilon, \text{man_on}/\text{trigger}, \text{auto_on}/\text{trigger}, \text{man_off}/\text{release}, \text{auto_off}/\text{release},$
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 $\text{man_on}/\text{trigger}.\text{man_off}/\text{release}, \text{man_on}/\text{trigger}.\text{auto_off}/\text{trigger},$
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T-Method

Every **TS** = P transition cover of M
is a complete test suite of $\mathcal{F}(M, I, O, \sim, \mathcal{D}_O)$

Proof:

Suppose M', M are not I/O equivalent.

Then there exists $\pi \in \Sigma^*$, $\sigma \neq \sigma' \in \Sigma$ such that $\pi.\sigma \in L(M)$ and $\pi.\sigma' \in L(M')$ with $\sigma_I = \sigma'_I$.

Let $q_0 \xrightarrow{\pi} q$, $q \xrightarrow{\sigma} q_1 \in h$, and $q \xrightarrow{\sigma'} q_1 \in h'$.

Since **TS** is a transition cover, there is $\tau \in \mathbf{TS}$ such that $q_0 \xrightarrow{\tau} q \in h$ and $\tau.\sigma \in \mathbf{TS}$.

Let $q_0 \xrightarrow{\tau'} q \in h'$ with $\tau_I = \tau'_I$.

Then $q_0 \xrightarrow{\tau'} q \xrightarrow{\sigma'} q_1 \in h'$ and $q_0 \xrightarrow{\tau} q \xrightarrow{\sigma} q_1 \in h$.

Since $\tau'_I.\sigma'_I = \tau_I.\sigma_I$ and $\tau'_O.\sigma'_O \neq \tau_O.\sigma_O$, we have $\tau.\sigma \notin L(M')$

Hence M' fails the test case $\tau.\sigma \in \mathbf{TS}$.

W-Method

M. P. Vasilevskii 1973 and Tsun S. Chow 1978

$\mathcal{F}(M, I, O, \leq, \mathcal{D}_m)$, fault model

$$\mathcal{D}_m = \{M' = (Q', q'_0, I, O, h') \in Sig \mid |Q'| \leq m\}$$

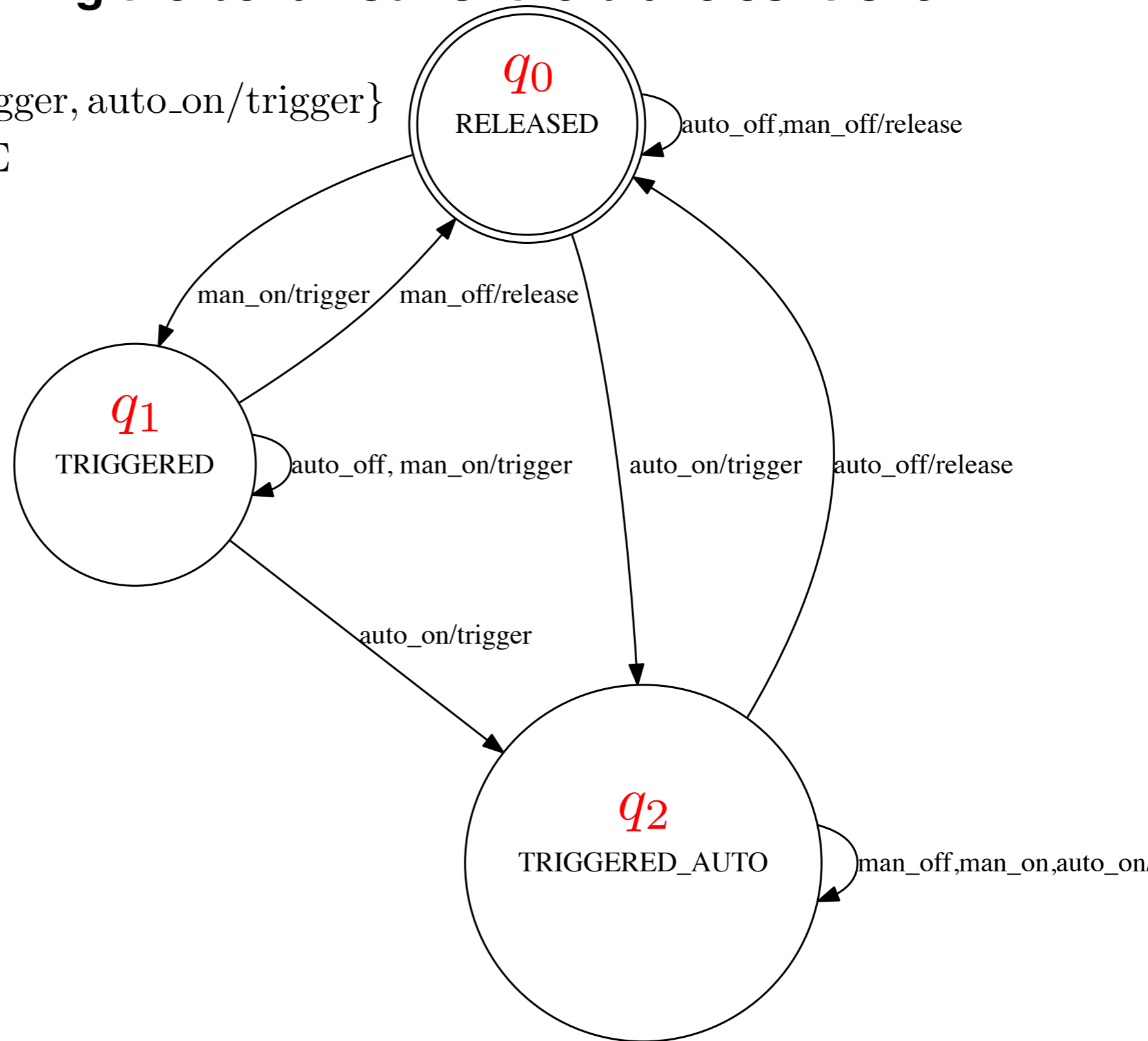
Characterization Set

Characterization set W of $M = (Q, q_0, I, O, h)$

- $W \subseteq \Sigma^*$ is a set of I/O sequences
- $\forall q_1 \neq q_2 \in Q, \exists \tau_1 \neq \tau_2 \in W : \tau_{1_I} = \tau_{2_I} \wedge \tau_i \in L(q_i), i = 1, 2$

Finite State Machine modelling the behaviour of the brake controller

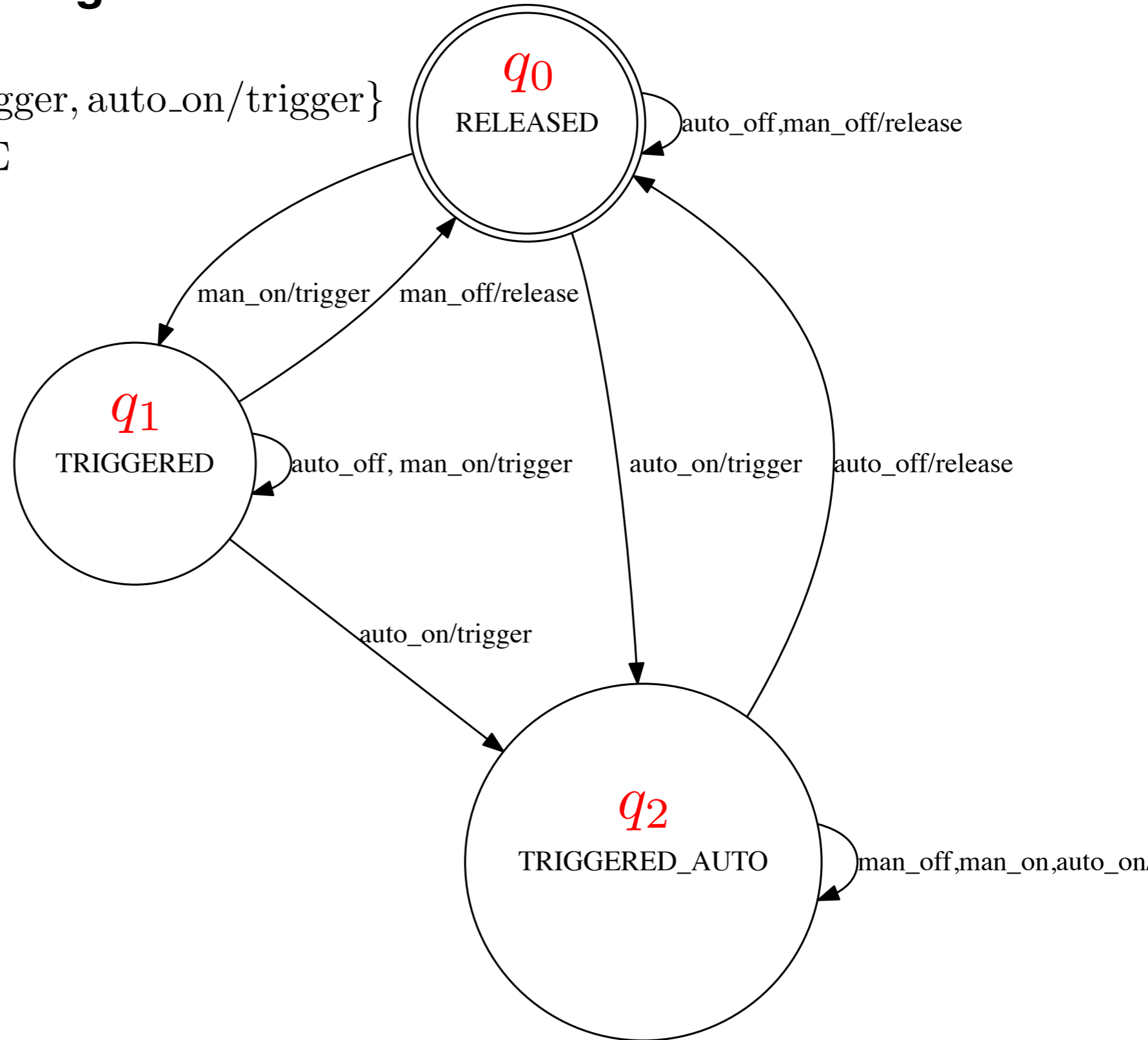
- state cover $V = \{\varepsilon, \text{man_on}/\text{trigger}, \text{auto_on}/\text{trigger}\}$
- transition cover $P = V \cup V \oplus \Sigma$



Finite State Machine modelling the behaviour of the brake controller

- state cover $V = \{\varepsilon, \text{man_on}/\text{trigger}, \text{auto_on}/\text{trigger}\}$
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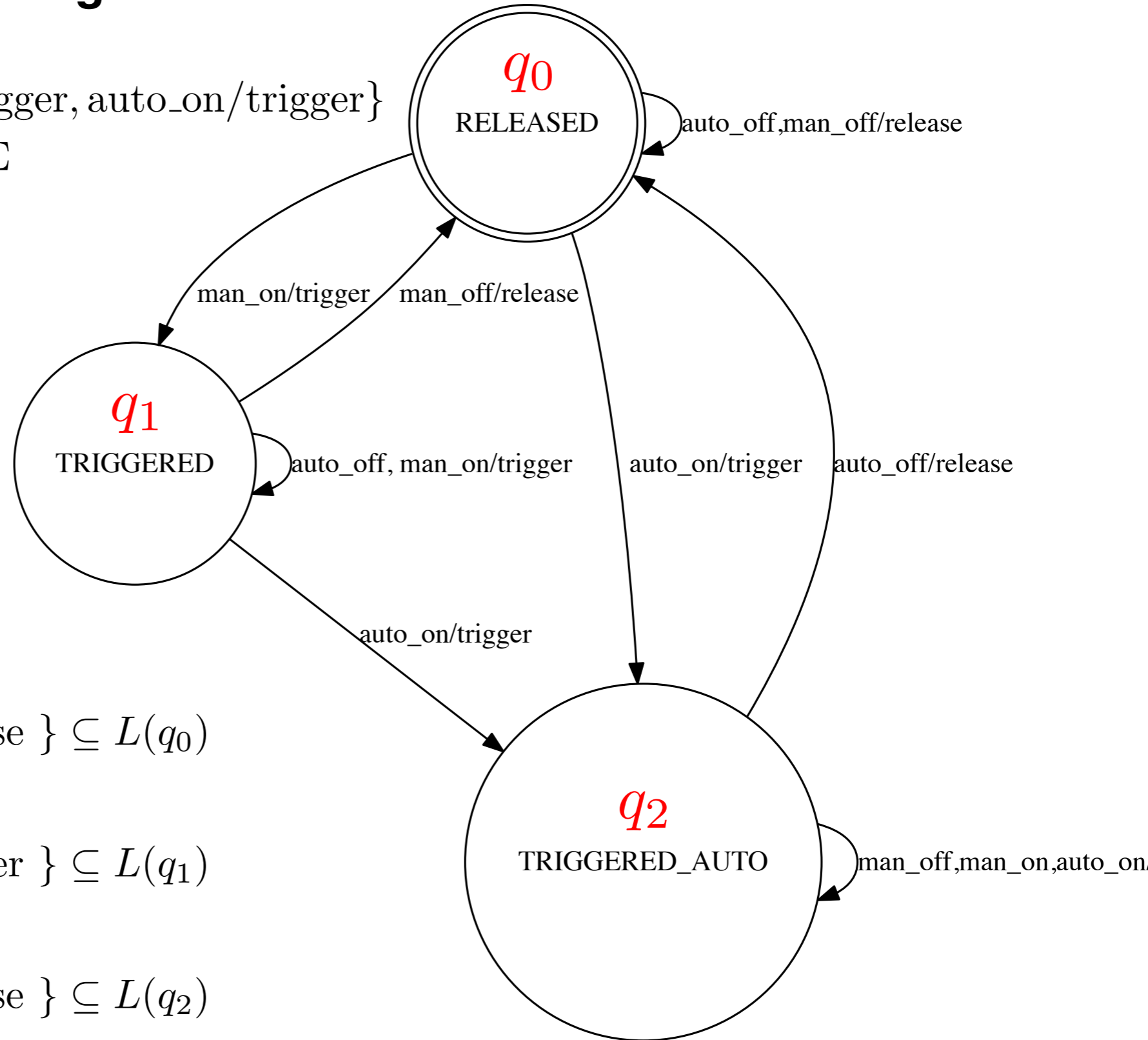
$q_0 \xrightarrow{\text{auto_off}/\text{release}} q_0$
 $q_1 \xrightarrow{\text{auto_off}/\text{trigger}} q_1$
 $q_2 \xrightarrow{\text{auto_off}/\text{release}} q_0$
 $q_0 \xrightarrow{\text{man_off}/\text{release}} q_0$
 $q_2 \xrightarrow{\text{man_off}/\text{trigger}} q_2$



Finite State Machine modelling the behaviour of the brake controller

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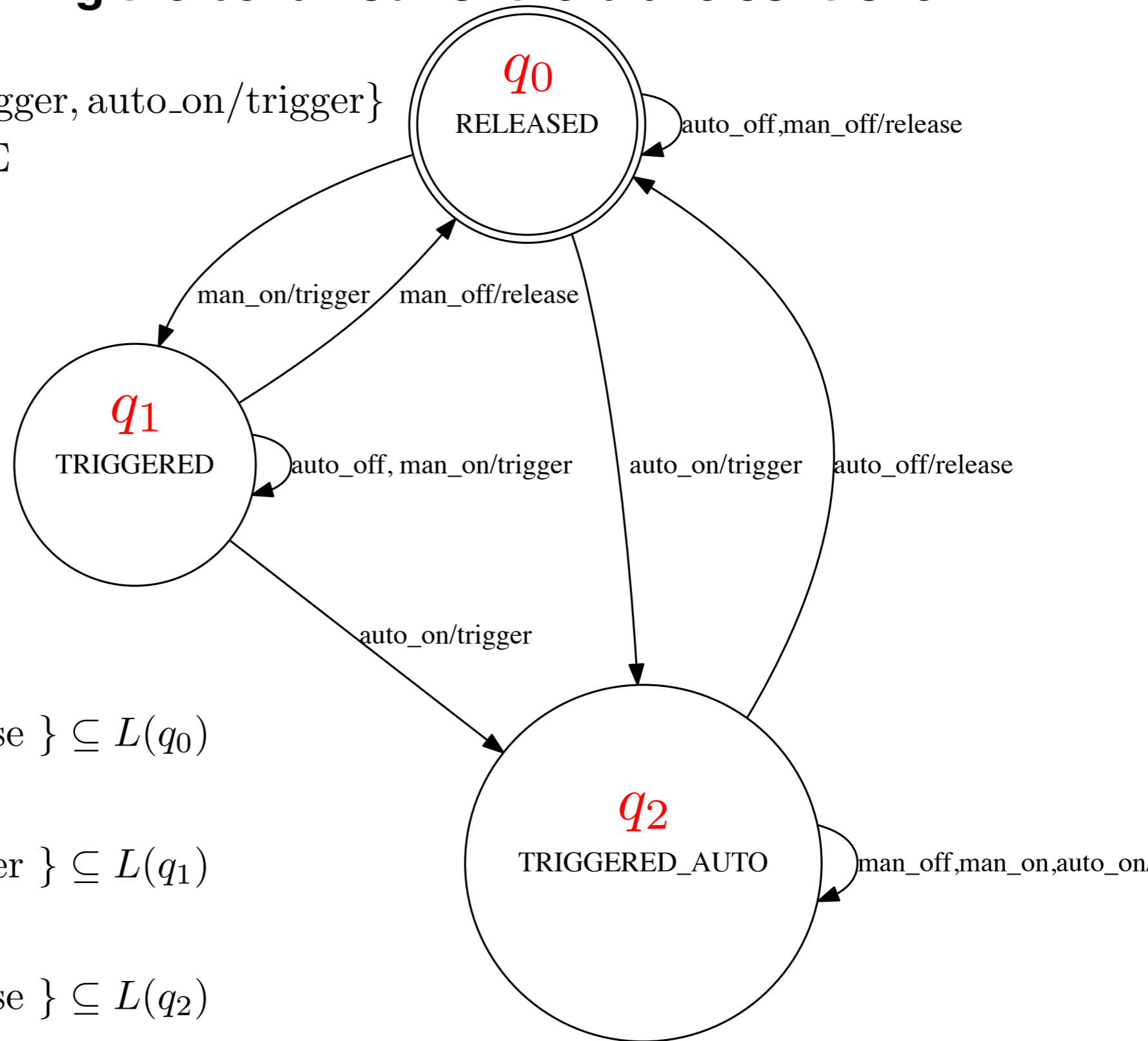
$q_0 \xrightarrow{\text{auto_off}/\text{release}} q_0$
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 $q_2 \xrightarrow{\text{auto_off}/\text{release}} q_0$
 $q_0 \xrightarrow{\text{man_off}/\text{release}} q_0$
 $q_2 \xrightarrow{\text{man_off}/\text{trigger}} q_2$



- $\{\text{man_off}/\text{release}, \text{auto_off}/\text{release}\} \subseteq L(q_0)$
- $\{\text{man_off}/\text{release}, \text{auto_off}/\text{trigger}\} \subseteq L(q_1)$
- $\{\text{man_off}/\text{trigger}, \text{auto_off}/\text{release}\} \subseteq L(q_2)$

Finite State Machine modelling the behaviour of the brake controller

- state cover $V = \{\varepsilon, \text{man_on}/\text{trigger}, \text{auto_on}/\text{trigger}\}$
- transition cover $P = V \cup V \oplus \Sigma$



- $\{\text{man_off}/\text{release}, \text{auto_off}/\text{release}\} \subseteq L(q_0)$

- $\{\text{man_off}/\text{release}, \text{auto_off}/\text{trigger}\} \subseteq L(q_1)$

- $\{\text{man_off}/\text{trigger}, \text{auto_off}/\text{release}\} \subseteq L(q_2)$

- characterization set $W = \{\text{man_off}/\text{trigger}, \text{man_off}/\text{release}, \text{auto_off}/\text{trigger}, \text{auto_off}/\text{release}\}$

W-Method

Every **TS** = $V \oplus \left(\bigcup_{i=0}^{m-n+1} \Sigma^i \right) \oplus W$
is a complete test suite of $\mathcal{F}(M, I, O, \sim, \mathcal{D}_m)$, $n = |Q|$

W-Method

Every $\mathbf{TS} = V \oplus \left(\bigcup_{i=0}^{m-n+1} \Sigma^i \right) \oplus W$
is a complete test suite of $\mathcal{F}(M, I, O, \sim, \mathcal{D}_m)$, $n = |Q|$

$$\mathbf{TS}_I = V_I \cdot \left(\bigcup_{i=0}^{m-n+1} I^i \right) \cdot W_I$$

Step 1.

$M = (Q, q_0, I, O, h), M' = (Q', q'_0, I, O, h') \in \text{Sig}.$

Suppose $\exists f : Q \rightarrow Q'$:

- $f(q_0) = q'_0$
- $(q_1, x, y, q_2) \in h \Rightarrow (f(q_1), x, y, f(q_2)) \in h'$

Then $L(M) = L(M')$.

Step 1.

$M = (Q, q_0, I, O, h), M' = (Q', q'_0, I, O, h') \in \text{Sig}.$

Suppose $\exists f : Q \rightarrow Q'$: homomorphism

- $f(q_0) = q'_0$
- $(q_1, x, y, q_2) \in h \Rightarrow (f(q_1), x, y, f(q_2)) \in h'$

Then $L(M) = L(M')$.

Proof of step 1.

Suppose $f : Q \rightarrow Q'$, $f(q_0) = q'_0$, is a homomorphism.

Then $\forall x_1 \dots x_k / y_1 \dots y_k \in L(M)$, $\exists q_i \in Q$, $i = 1, \dots, k$:

$q_0 \xrightarrow{x_1/y_1} q_1 \xrightarrow{x_2/y_2} \dots \xrightarrow{x_k/y_k} q_k$ and

$q'_0 = f(q_0) \xrightarrow{x_1/y_1} f(q_1) \xrightarrow{x_2/y_2} \dots \xrightarrow{x_k/y_k} f(q_k)$

Then $x_1 \dots x_k / y_1 \dots y_k \in L(M')$ and $L(M) \subseteq L(M')$.

Since M, M' are input complete and deterministic,
we have $L(M') = L(M)$.

Step 2.

$M' = (Q', q'_0, I, O, h') \in \mathcal{D}_m.$

Suppose M' pass $V \oplus \bigcup_{i=0}^{m-n} \Sigma^i \oplus W$

Then $V \oplus \bigcup_{i=0}^{m-n} \Sigma^i$ is a state cover of M' and

$V \oplus \bigcup_{i=0}^{m-n+1} \Sigma^i$ is a transition cover of M' .

Proof of step 2.

1. $M' \underline{\text{pass}}(V \cdot \bigcup_{i=0}^{m-n+1} \Sigma^i \cdot W) \cap L(M)$
 $\Rightarrow (V \cdot \bigcup_{i=0}^{m-n+1} \Sigma^i \cdot W) \cap L(M) = (V \cdot \bigcup_{i=0}^{m-n+1} \Sigma^i \cdot W) \cap L(M')$
 $\Rightarrow V \oplus \bigcup_{i=0}^{m-n+1} \Sigma^i \oplus W = V \oplus' \bigcup_{i=0}^{m-n+1} \Sigma^i \oplus' W$
2. $|\{q'_0\text{-after-}\pi \mid \pi \in V\}| \geq n$
3. $\{q'_0\text{-after-}\pi \mid \pi \in V \oplus \bigcup_{i=0}^{m-n} \Sigma^i\} = Q'$

Proof of $|\{\{\mathbf{q}'_0\text{-after-}\pi \mid \pi \in \mathbf{V}\}\}| \geq \mathbf{n}$

Let

- $\pi_0 = \varepsilon$
- $\{\pi_i \mid i = 0, \dots, n - 1\} \subseteq V$ state cover of M
- $q_0 \xrightarrow{\pi_i} q_i$
- $q'_0 \xrightarrow{\pi_i} q'_i$

Then $i \neq j \Rightarrow q'_i \neq q'_j$:

1. $q_i \neq q_j \Rightarrow \exists \tau_i \neq \tau_j \in W : \tau_{i_I} = \tau_{j_I}, \tau_i \in L(q_i), \tau_j \in L(q_j)$

(W is a characterisation set of M)

2. $\tau_i \in L'(q'_i), \tau_j \in L'(q'_j) \wedge \tau_{i_I} = \tau_{j_I}$

3. M' is deterministic $\Rightarrow L'(q'_i) \neq L'(q'_j) \Rightarrow q'_i \neq q'_j$

Hence $n = |\{q'_0, \dots, q'_{n-1}\}| \leq |\{q'_0\text{-after-}\pi \mid \pi \in V\}|$

Step 3.

$M' = (Q', q'_0, I, O, h') \in \mathcal{D}_m.$

Suppose M' pass $V \oplus \bigcup_{i=0}^{m-n+1} \Sigma^i \oplus W.$

Then $\exists f : M' \rightarrow M$ homomorphism.

Proof of step 3.

Let

- $Q' =: \{q'_0, \dots, q'_{m-1}\}$
- $\{\varepsilon\} \cup \{\pi_i \mid i = 1, \dots, m-1\} \subseteq V \oplus \bigcup_{i=0}^{m-n} \Sigma^i$,
a state cover of M' .
- $f : Q' \rightarrow Q$, $f(q'_0) = q_0$ and $f(q'_i) = q_0$ -after- π_i .

Proof of step 3.

Let

- $Q' =: \{q'_0, \dots, q'_{m-1}\}$
- $\{\varepsilon\} \cup \{\pi_i \mid i = 1, \dots, m-1\} \subseteq V \oplus \bigcup_{i=0}^{m-n} \Sigma^i$,
a state cover of M' .
- $f : Q' \rightarrow Q$, $f(q'_0) = q_0$ and $f(q'_i) = q_0\text{-after-}\pi_i$.

Then $f(q)$ is the unique state of M such that
 $\pi \in L(q') \Leftrightarrow \pi \in L(f(q)), \forall \pi \in W$

Proof of step 3.

Let

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Then $f(q)$ is the unique state of M such that

$$\pi \in L(q') \Leftrightarrow \pi \in L(f(q)), \forall \pi \in W$$

Then

$$x/y \in L(q'_i) \stackrel{m-n}{\Leftrightarrow} x/y \in L(q_i), \quad \forall x/y \in \Sigma$$
$$[q_0, q'_0 \text{ pass}(V \oplus \bigcup_{i=0} \Sigma^i) \oplus \Sigma]$$

$$\tau \in L(q'_i) \stackrel{m-n}{\Leftrightarrow} \tau \in L(q_i), \quad \forall \tau \in W$$
$$[q_0, q'_0 \text{ pass}(V \oplus \bigcup_{i=0} \Sigma^i) \oplus W]$$

$$(x/y).\tau \in L(q'_i) \stackrel{m-n}{\Leftrightarrow} (x/y).\tau \in L(q_i), \quad \forall x/y \in \Sigma, \tau \in W$$
$$[q_0, q'_0 \text{ pass}(V \oplus \bigcup_{i=0} \Sigma^i) \oplus \Sigma \oplus W]$$

$$\begin{array}{ccccc}
 q'_0 & \xrightarrow{\pi_i} & q'_i & \xrightarrow{x/y} & q' \\
 \downarrow f & & \downarrow f & & \downarrow f \text{ ?} \\
 q_0 & \xrightarrow{\pi_i} & q_i & \xrightarrow{x/y} & q
 \end{array}$$

$$\begin{aligned}
 \pi \in L(q') &\Leftrightarrow \pi \in L(q), \forall \pi \in W \\
 \Rightarrow f(q') &= q
 \end{aligned}$$

Wp-Method

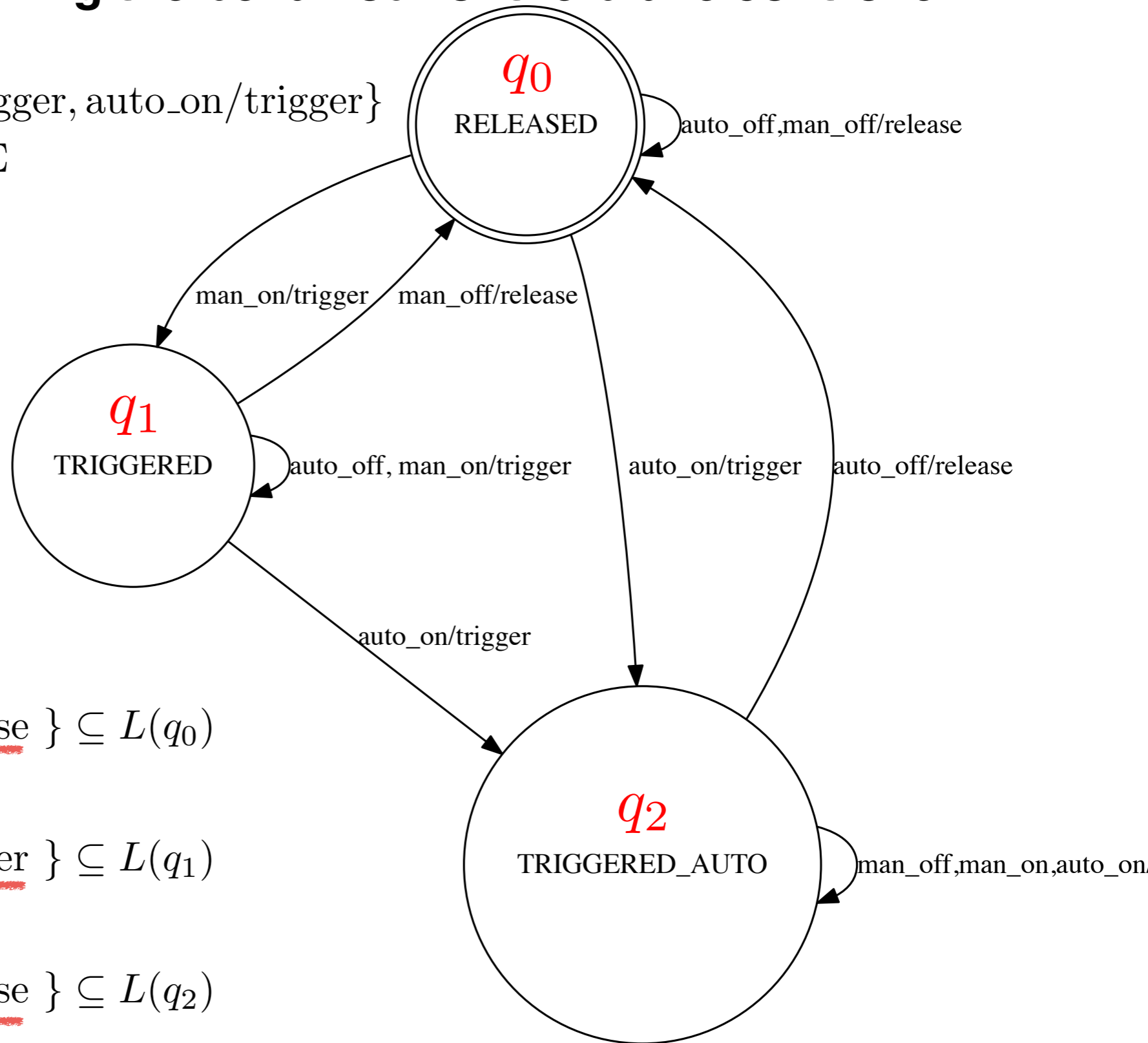
$\mathcal{F}(M, I, O, \leq, \mathcal{D}_m)$, fault model

$$\mathcal{D}_m = \{M' = (Q', q'_0, I, O, h') \subseteq Sig \mid |Q'| \leq m\}$$

1. V a state cover of M .
2. $P = V \oplus (\{\varepsilon\} \cup \Sigma)$ a transition cover of M
3. $R = P \setminus V$.
4. W a characterisation set of M .
5. $\{W_0, \dots, W_{n-1}\}$ *state identification sets* of M , such that
 - $W_i \subseteq \text{pref}(W)$ for $i = 0, \dots, n - 1$.
 - W_i distinguishes q_i from all other states in Q .

Finite State Machine modelling the behaviour of the brake controller

- state cover $V = \{\varepsilon, \text{man_on}/\text{trigger}, \text{auto_on}/\text{trigger}\}$
- transition cover $P = V \cup V \oplus \Sigma$



- $\{\text{man_off}/\text{release}, \text{auto_off}/\text{release}\} \subseteq L(q_0)$
 W_0

- $\{\text{man_off}/\text{release}, \text{auto_off}/\text{trigger}\} \subseteq L(q_1)$
 W_1

- $\{\text{man_off}/\text{trigger}, \text{auto_off}/\text{release}\} \subseteq L(q_2)$
 W_2

- characterization set $W = \{\text{man_off}/\text{trigger}, \text{man_off}/\text{release}, \text{auto_off}/\text{trigger}, \text{auto_off}/\text{release}\}$

Wp-Method

- $Wp_1 = V \oplus \left(\bigcup_{i=0}^{m-n} \Sigma^i \right) \oplus W$
- $Wp_2 = R \oplus \Sigma^{m-n} \oplus \{W_0, \dots, W_{n-1}\}$
- **TS** = $Wp_1 \cup Wp_2$
is a complete test suite of $\mathcal{F} = (M, I, O, \sim, \mathcal{D}_m)$.

$$U \oplus \{W_0, \dots, W_{n-1}\} =$$

$$\bigcup_{\pi \in U \wedge q_i = q_0 \text{-after-}\pi} \{\pi\} \cdot W_i$$

Wp-Method

- $Wp_1 = V \oplus \left(\bigcup_{i=0}^{m-n} \Sigma^i \right) \oplus W$
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Further Reading

1. Publications of Jan Peleska, Wen-ling Huang, and their co-authors. http://www.informatik.uni-bremen.de/agbs/jp/jp_papers_e.html
2. ERTMS/ETCS SystemRequirements Specification, Chapter 3, Principles, volume Subset-026-3, Issue 3.4.0 (2015), available under <http://www.era.europa.eu/Document-Register/Pages/Set-2-System-Requirements-Specification.aspx>
3. Nancy Leveson. SafeWare: System Safety and Computers. Addison Wesley 1995.
4. Neil Storey. Safety-critical Computer Systems. Addison-Welly, 1996.